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Net Zero Investment Portfolios

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Abstract

The emergence of net zero emissions policies is currently one of the most important topics among asset owners and managers. It considerably changes portfolio allocation and the investment framework of both passive and active investors. The academic literature generally concludes that implementing net zero portfolios and sustainable investing is not costly. Moreover, some investors have chosen to implement highly dynamic decarbonization pathways with a continuous reference to business-as-usual benchmarks. The goal of this actuarial thesis is to participate in the debate on climate investing by showing that it is not a free lunch. Net zero investment portfolios involve some substantial costs in terms of tracking, diversification, and liquidity risks. Furthermore, the reference to business-as-usual benchmarks is not always relevant because climate investing in a net zero framework is not a simple extension of traditional investing.

The decarbonization pathway requires the net zero emissions scenario to be defined. Transforming this absolute scenario into an intensity-based scenario is not straightforward because it involves a carbon budget. Once the scenario is established, it is important to assess the metrics that capture the different dimensions of a net zero emissions policy, particularly, the self-decarbonization and the green intensity of issuers. Then we can combine these different figures to define the objective function involved in optimizing net zero portfolios by considering the asset class. For instance, bond portfolios and equity portfolios are not constructed in the same way. The objective of this integrated approach is to deal with the multi-faceted dimensions of net zero investing. Another method establishes a core-satellite portfolio, where decarbonization and transition dimensions are segregated.

The results of this dissertation show that net zero investing goes beyond the simple exercise of dynamic decarbonization. Compared to a business-as-usual benchmark, the tracking error cost may be relatively high, especially for equity portfolios. Moreover, the diversification risk is critical for equities and bonds because we see significant deformation of investment universes. These results indicate that climate investing is not just a tilt of business-as-usual or traditional investing. Since it is a new investment framework and not another new thematic, asset owners and managers must move away from the traditional approach, which considers that the reference portfolio is the business-as-usual benchmark. Of course, this situation is transitory until the world is on the right track to becoming a net zero economy, but at that time, we will again observe a convergence between business-as-usual and climate investing, and a growing correlation between the market and net zero portfolios.

Keywords: Climate change, net zero emissions scenario, decarbonization, transition, greenness.

JEL Classification: G11, Q5.

Résumé

L'émergence de politiques net zéro est un sujet majeur parmi les différents acteurs de la gestion d'actif. Elle modifie considérablement l'allocation des portefeuilles et le cadre d'investissement des investisseurs actifs et passifs. La littérature académique conclut généralement que la mise en place de portefeuilles net zéro et de critères de durabilité n'est pas coûteuse. En outre, certains investisseurs ont choisi de suivre des trajectoires de décarbonation très dynamiques, en se référant constamment à un portefeuille de référence "*business-as-usual*". L'objectif de ce mémoire est de participer au débat sur l'investissement climatique, en insistant sur sa complexité. Les portefeuilles d'investissement net zéro impliquent ainsi des coûts significatifs en termes de risques de *tracking*, de diversification et de liquidité. Par ailleurs, l'utilisation de portefeuilles *business-as-usual* en guise de *benchmark* n'est pas toujours pertinente car l'investissement climatique dans un cadre net zéro n'est pas une simple extension de l'investissement traditionnel.

Le choix de la trajectoire de décarbonation nécessite de définir un scénario d'émissions net zéro. Or, les indices de références européens proposent des trajectoires basées sur la réduction de l'intensité carbone. Le passage d'un scénario en émissions absolues à un scénario basé sur l'intensité n'est alors pas aisé car il implique un budget carbone. Une fois le scénario d'émission établi, il est nécessaire d'identifier les métriques permettant de saisir les différentes dimensions d'une politique net zéro, en particulier l'auto-décarbonation et l'intensité verte des émetteurs. Nous pouvons ensuite combiner ces indicateurs afin de définir le problème d'optimisation des portefeuilles net zéro, selon la classe d'actif considérée. En effet, les portefeuilles d'actions et les portefeuilles obligataires ne sont pas construits selon la même méthodologie. L'objectif de cette approche, que nous appelons intégrée, est alors de prendre en considération les multiples dimensions de l'investissement net zéro. Une autre méthode consiste à établir un portefeuille coeur-satellite, où les dimensions de décarbonation et de transition sont séparées.

Les résultats de ce mémoire montrent que l'investissement net zéro va au-delà du simple exercice de décarbonation. Par rapport à un portefeuille de référence *business-as-usual*, le coût en *tracking error* peut s'avérer relativement élevé, notamment dans le cas des portefeuille actions. De surcroît, le risque de diversification est critique tant pour les actions que pour les obligations, du fait de la déformation importante des univers d'investissement. Ces résultats suggèrent que l'investissement climatique n'est pas une simple extension de l'investissement traditionnel. Puisqu'il s'agit d'un nouveau cadre d'investissement, et non simplement d'une nouvelle thématique, les gestionnaires d'actifs doivent s'éloigner de l'approche traditionnelle et des portefeuilles *business-as-usual*. Cette situation est bien entendu transitoire, jusqu'à ce que l'économie rejoigne la trajectoire d'un monde neutre en carbone. Nous devrions alors à ce moment observer une convergence entre le *business-as-usual* et l'investissement climatique ainsi qu'une corrélation croissante entre le marché et les portefeuilles net zéro.

Keywords: Changement climatique, Scénario d'émissions net zéro, neutralité carbone, décarbonation, transition, actif vert.

JEL Classification: G11, Q5.

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1 Introduction

Climate risk is the biggest challenge to humanity in the 21st century. Indeed, climate change implies higher temperatures that increase the likelihood of extreme weather events and impact living patterns. Beyond the direct effect on natural hazards, climate change may also result in a new economic order because of the transition to a low-carbon economy. Physical and transition risks question the resilience of the financial system to climate-related risks. This explains why climate change has become the top priority for financial institutions, supervisors, and policymakers. The asset management industry is primarily concerned because of the transmission channel on asset prices. Therefore, portfolio decarbonization, temperature alignment, net zero investment, and Paris-aligned benchmarks are the day-to-day reality for both asset owners and managers. Since 2014, interest in climate-related financial risks has been boosted by the development of ESG investing in Europe (Bennani *et al.*, 2018; Drei *et al.*, 2019). While environmental issues have lagged behind social issues during the Covid-19 crisis, the net zero carbon race and the Glasgow COP 26 event have recently changed the equation, and climate risk is now the hottest topic in asset management. This explains why climate investing is the new investment theme for asset owners and managers. Initially, this mainly involved decarbonizing portfolios, constructing low-carbon indices, and investing in climate-related securities such as green bonds. However, the concept of net zero has accelerated the scope of climate investing and we may wonder if it has profoundly changed its nature. Before the Covid-19 crisis, climate investing could be viewed as an investment strategy or a thematic strategy like factor investing, smart beta, quality investing, or a growth strategy. But the proliferation of net zero alliances[1] (GFANZ, NZAOA, NZAM, NZBA, etc.) and their commitments imply new dynamics in climate investing that cannot be compared to the dynamics of a thematic investment. As such, considering net zero portfolios as a tilt of a business-as-usual portfolio is not obvious. This was not the case with low-carbon portfolios and indices, because a low-carbon strategy consists in removing issuers with the highest carbon footprints. With net zero portfolios, it is another story because the goal is also to green the economy, and, here, there is a long way to go (Fankhauser *et al.*, 2022; Philipponnat, 2022). For instance, focusing on equities and corporate bonds, Alessi and Battiston (2022) estimated "*a greenness of about 2.8% for EU financial markets*" according to the existing European green taxonomy (European Commission, 2020, 2021a,b). The current greenness of the economy and the financial market is therefore a real challenge for net zero investment policies.

Net zero investing challenges

If we read reports from international bodies on the feasibility of net zero emissions by 2050, we notice that the decarbonization pathway of the net zero scenario has two statuses. It is the exogenous pathway that the economy must follow to limit the probability of reaching 1.5°C. However, it is not the solution to the problem, because we have to take some action to reach this objective. If the world and its economic stakeholders make the right decisions, the decarbonization pathway then becomes the endogenous pathway that the economy can follow to limit the probability of reaching 1.5°C. What are these right decisions? They are very diverse, and the purpose of this dissertation is not to list them, but they share a common feature. Indeed, they all require massive financing and involve new investments:

> "*Capital spending on physical assets for energy and land-use systems in the net-zero transition between 2021 and 2050 would amount to about* \$275 *trillion, or* \$9.2

[1] GFANZ = Glasgow Financial Alliance for Net-Zero, NZAOA = Net Zero Asset Owner Alliance, NZAM = Net Zero Asset Managers initiative, NZBA = Net Zero Banking Alliance.

trillion per year on average, an annual increase of as much as \$3.5 *trillion from today*" (McKinsey, 2022, page viii).

This figure of \$3.5 trillion is approximately equal to 1/2 of global corporate profits, 1/4 of total tax revenue, or 4.1% of world GDP. Therefore, the gap between current and expected investments is huge. It does not only concern the private sector, but that should still drive us to better define a net zero carbon commitment. Indeed, when asset owners and managers speak about net zero investing, they mainly focus on portfolio decarbonization. Reducing the portfolio's carbon footprint is important, but net zero investing goes beyond a simple portfolio decarbonization exercise. As shown by the McKinsey report, the real challenge of net zero is the transition dimension, in particular how to finance the transition to a low-carbon economy.

Building a net zero investment portfolio is more complex than building a decarbonized portfolio, because the objective function encompasses at least two goals: decarbonizing the portfolio and financing the transition. Moreover, the decarbonization dimension is no longer static. It becomes dynamic. Most investors have solved this issue by considering a time-varying reduction rate. In this case, we could wonder whether the decarbonization dimension of net zero investing could be summarized by a sequence of decarbonization rates. Indeed, if net zero investing consists in building successive independent portfolios, there is no mechanism that respects the endogenous aspect of the decarbonization pathway. In particular, if the time-varying decarbonization is only due to the rebalancing process, it is clear that the portfolio cannot claim to be net zero. Indeed, the endogenous aspect of the decarbonization pathway implies the self-decarbonization of the portfolio. Therefore, we must introduce an incentive mechanism to reach a minimum level of self-decarbonization. The objective of carbon temperature ratings is precisely to assess the capacity of an issuer to be aligned with a carbon emissions scenario. Carbon temperature can be viewed as a synthetic scoring system based on the $\mathcal{PAC}$ framework (Le Guenedal *et al.*, 2022), which measures the issuer's (past) participation, ambition and credibility. Since a rating system of carbon temperature is often perceived as a black box, we may consider a simplified approach that is more transparent. For instance, we can use net zero targets that are approved and validated by a third party. By using a linear interpolation model, we can compute the yearly self-decarbonization rate of issuers and deduce the self-decarbonization level of portfolios. This simple approach is limited for two reasons. First, the data are not homogeneous because targeted dates and scopes could be different. Second, the self-decarbonization cannot be computed for issuers without net zero engagement or validation. Another approach consists in focusing on the first pillar, which is participation. Indeed, participation is a technical term used to identify past self-decarbonization. This explains that carbon trends and carbon momentum measures are very important metrics for a net zero investor. This is a way to introduce a dynamic approach to the carbon footprint and to go beyond the current level, which is a poor estimate of the issuer's finish line and an even poorer one of how quickly the issuer will get there.

Besides net zero carbon metrics, the portfolio manager also needs net zero transition metrics to assess the greenness of the portfolio. Therefore, the green intensity is the equivalent of the carbon intensity for the transition dimension. One of the issues is the choice of the right metric. Indeed, there are many metrics and a lack of exhaustive data. Le Guenedal and Roncalli (2022) reported some of them, but most of the time they are sector-specific, biased, difficult to compute or not meaningful for all issuers. A typical example is the amount of avoided emissions, since it is not easy to define a reference for each product. This explains why the concept of green revenues has emerged and has been developed over the last few years. Once a green taxonomy is defined, green revenues can be easily computed using detailed income statements. In three years, green revenue share has become the main factor when computing a green intensity score. Nevertheless, this metric is relatively young, which explains why we do not have enough historical data to perform a dynamic analysis. An alternative is to use green capital expenditures (capex), green operational expenses (opex) or green R&D expenses, but they are under development, implying that these metrics will not be available before 2024.

Building a net zero portfolio is also not an easy task because a financial investment cannot reach net zero by itself. Only an economy, a region or a group of industries can reach net zero. Indeed, CO_2 emissions can be comprehensively measured for a relatively closed system, but not for an open system. This implies integrating scope 3 emissions in order to include the CO_2 emissions of the entire supply chain. This is another difference with low-carbon portfolios. At the same time, we know that scope 3 emissions data are of poor quality. Nevertheless, we face a critical situation where we do not have time and we have no choice. As such, the definition of a net zero investment strategy is not fixed and stabilized since we are using more of a learning-by-doing approach than a mature model. Therefore, net zero processes will evolve in the future as new metrics are adopted and data quality improves. In fact, the current situation could be transitory and may be explained because the economy's pathway is far from net zero. The consequence is the huge gap between market and net zero portfolios. Nevertheless, we believe that this situation will improve in the long run with the transition to a net zero economy, and we will observe a convergence between business-as-usual and net zero investing. In the meantime, net zero investing is a true test for ESG investors with strong ethical convictions. In the short run, the world economy is far from being on the right track and the current energy crisis is a new factor that challenges our ability to keep global warming below 1.5°C. The short-term risk is that the discrepancy between business-as-usual portfolios and net zero portfolios increases, in particular if the transition to a low-carbon economy is delayed. For a net zero portfolio, this is a micro-economic risk, but for the asset management industry, this is a macro-economic risk. Indeed, the high commitment of net zero alliances implies a large investment universe of net zero assets. However, the current investment universe is relatively small in terms of green or transition assets. This implies that the financial market and the issuers must become sufficiently green very quickly. Otherwise, the gap between traditional and climate investing would widen.

This dissertation is organized as follows. Regulatory framework and climate risks definition are introduced through Section Two. In Section Three, we introduce the concept of a net zero emissions scenario, which is a physical concept based on carbon budgets. We compare it to the financial concept of a decarbonization pathway based on the carbon intensity metric, and we also illustrate the relationships between emission-based and intensity-based scenarios. Section Four is dedicated to net zero metrics and contains two parts. The first part reviews the metrics associated with the decarbonization dimension. After studying static measures of carbon footprint, we consider dynamic measures that are related to the self-decarbonization aspect. In particular, we focus on the carbon momentum metric. The second part deals with the transition dimension. After a discussion on green taxonomy, we introduce static and dynamic measures of greenness such as green revenues and green capex. Section Five recalls some of the theoretical foundations of portfolio optimization. This sections introduces the Markowitz' mean-variance optimization in presence of a benchmark and discusses risk management metrics for equity and bond portfolios. The construction of net zero investment portfolios is discussed in Section Six. First, we analyze the impact of portfolio decarbonization in terms of tracking risk, sector allocation and transition metrics. We consider both equity and bond portfolios and show that the results are similar. Second, we present the integrated approach of net zero investing, which involves defining a unique optimization problem by considering all the aspects of the transition dimension. This implies adapting the original problem of portfolio decarbonization by adding many constraints. In this case, the results on equity portfolios differ from those on bond portfolios if we focus on tracking risk. Nevertheless, the results are similar in the two asset classes when we consider diversification and liquidity risks. In Section Six, we also present an alternative method for building net zero investment portfolios by using a core-satellite approach. Finally, Section Seven offers some concluding remarks.

2 Climate risk and regulation

The Network of Central Banks and Supervisors for Greening the financial System (NGFS), launched at the Paris One Planet Summit on December 12, 2017, is a group of volunteer central banks and supervisors wishing to share best practices and contribute to the development of climate change financial risk management. The stakes in the face of global warming are immense and the financial sector has a decisive role to play in financing the energy transition to a decarbonized economy. Regulators can intervene in two ways:

- contributing to the creation of an environment conducive to the ecological transition, in particular by monitoring the commitments of financial institutions and ensuring the transparency of their exposure ;
- ensuring the protection of financial institutions against climate risks in order to guarantee financial stability by verifying that institutions have identified these risks and put in place methods dedicated to their management.

This secont point has recently been extensively investigated, notably through an exercise of climate stress-testing led by the Autorité de Contrôle Prudentiel et de Résolution (2020) (ACPR).

2.1 ACPR pilot exercise

The main role of this exercise is to make banks and insurance companies aware of the risks they will have to face with climate change so that they integrate this long-term component into their governance and strategy. The goal is to highlight the vulnerability of institutions to different climate scenarios. These scenarios suggests more or less brutal transitions in order to fight against greenhouse gas emissions, taking as a reference the trajectory drawn by the Paris Agreements aiming at limiting global warming to less than 2°C. By studying several scenarios, the stress test makes it possible to measure the cost of a trajectory that deviates from the policies defined by the COP21 agreements. Before investigating these scenarios, the pilot exercise declines climate risks in two categories.

2.1.1 Transition risk

Transition risk is the risk associated with the change in behavior of economic and financial agents required to reduce greenhouse gas emissions. Indeed, the reduction of emissions will have an impact on all the actors of the economy by modifying the value of financial assets and the profitability of companies. According to the ACPR, the sectors sensitive to transition risk are estimated at nearly 12% of banking assets and nearly 10% of the assets of insurance companies.

2.1.2 Physical risk

Physical risk measures the direct impact of climate change on people and property. This risk is generally divided into two categories:

- chronic risks (sea level rise, increase in average temperature, etc., which can progressively deteriorate the activity of an area) ;
- the risk of the occurrence of extreme climatic events, the damage to which can lead to the destruction of physical assets (e.g. real estate).

In its climate stress test exercise, the ACPR considers the impact of the physical risk on the frequency and costs associated with extreme weather events for insurance companies, in particular related to floods, droughts and cyclones (for overseas departments).

2.2 Regulatory framework

Climate risk has thus been a subject of rising interest within the finance and insurance industry. Literature abounds more and more of studies that discuss climate change and its impact on the insurance business through various channels. For instance, some actuarial thesis focused on the impact of climate change on mortality risk (Germain, 2022) whereas others discussed the change of floods (Boyeau, 2022), hails or even hurricanes risks induced by climate uncertainty. The common feature of these studies is their focus on risks insurers might face through the realization of extreme weather conditions or events because of climate change, commonly called physical risk. As suggested by the ACPR pilote exercise of may 2021, insurance actors have a traditional knowledge about how to deal with these risks, thanks to their historical Cat Nat departments for example. However, fewer works were conducted regarding the transition risk. Economic and financial agents will indeed have to change their behavior in order to participate in the transition to a low-carbon economy and this may affect the market portfolios of insurers and asset managers.

Alongside the increasing number of regulations in the asset management industry, insurance supervisory authorities have begun to require the assessment of these risks within insurance companies. Following a public consultation on how to integrate the customer's sustainability preferences under the Insurance Distribution Directive (IDD), the European Commission charged the EIOPA to propose the necessary changes for the integration of ESG preferences in the following regulations : investment fund regulations, Solvency 2 concerning the prudential framework for insurers, MIFID 2 and IDD regarding regulations for the distribution of financial products.

Solvency 2 EIOPA specifies that sustainability risks, and in particular climate change, will definitely impact the activity of insurance companies. As such, it considers that insurers, having to act according to the PPP, must integrate these risks in the management of their activity, all sectors included.

The consultation led to several key points regarding Solvency 2 :

- integration of sustainability risks into risk management ;
- integration of sustainability risks into the prudent person principle (PPP) ;
- integration of sustainability risks in the actuarial function ;
- integration of sustainability risks in the second pillar (ORSA).

IDD While the Insurance Distribution Directive originally aimed at regulating how insurance products are designed and distributed in the EU to make sure sold products meet clients' needs, climate risk has become more and more prevalent in the minds of investors. Hence, integrating customers climate preferences came as a natural extension of IDD.

The new legislation aims to ensure that retail investors can "*invest and save sustainably and facilitate their participation in the transition to a low-carbon, more sustainable, resource-efficient and circular economy in line with the Sustainable Development Goals, as insurance intermediaries and insurers have to recommend Investment-Based Insurance Products (IBIPs) that meet the sustainability preferences of their customers or potential customers, if they have such preferences:*".

IDD will hence introduce important changes about the way in which those sustainability preferences of the individual customer need to be taken into account when insurers and insurance intermediaries provide advice on IBIPs as part of the so-called suitability assessment.

MiFID 2 and sustainable preferences The financial industry has been all the more impacted as sustainabilty preferences saw the rise of ESG investing throughout the last decade[2]. MiFID is the Markets in Financial Instruments Directive (2004/39/EC). It has been applicable

[2] see Roncalli (2023) for a deeper understanding of financial regulation in place and incoming.

across the European Union to investment advice and portfolio management activity since November 2007. Its aim is to standardize practices across the EU for investment services and activities and to ensure a high degree of harmonised protection for investors in financial instruments. MiFID II is a revised version of the original MiFID and came into force in 2018. It covers organisational requirements for investment firms, regulatory reporting to avoid market abuse, OTC trading, transparency of costs, etc. Concerning investor protection, financial advisors must make a suitability and appropriateness assessment for individual portfolio management or advice regarding financial instruments. This implies financial advisers must obtain information from the client before it provides investment advice or individual portfolio management. The MiFID II Suitability Test includes questions about investors' knowledge and experience, their financial position, and their investment objectives. In September 2022, ESMA has published its guidelines on integrating ESG risks and factors in MiFID II (European Securities and Markets Authority , 2022). There are two main consequences:

1. Integration of sustainability preferences to define the suitable product;
2. Integration of ESG criteria in the product governance.

The first point ensures that the product is in line with investors' values when providing financial advice and portfolio management services. This implies a new version of the suitability and appropriateness assessment (profiling questionnaire, suitability test, adequacy report). The second point covers the product offering of FMPs. Indeed, manufacturers and distributors must specify their target markets and the sustainability-related objectives with which the product is compatible. "Sustainability preferences" is the key concept when selling an ESG product. If the client has any sustainability preferences (yes/no), it has to choose one or a combination of the criteria below:

1. Minimum percentage in environmentally sustainable investments aligned to the EU Taxonomy;
2. Minimum percentage invested in sustainable investments as defined in the SFDR (Sustainable Finance Disclosure, Articles 8 and 9).
3. Quantitative/qualitative elements of principal adverse impacts defined by the client.

Once the choice is done, the financial adviser can sell a product to the client only after ensuring that the product matches the sustainability preferences of the client. On this last aspect, the key question of the EIOPA's consultation is which market standards or labels should be used to characterize ESG and sustainable products and how to define such labels.

Remark 1. *The integration of sustainability preferences is not limited to financial investment products and MiFID II as highlighted above. It also applies to insurance-based investment products and the Insurance Distribution Directive (IDD).*

3 Net zero emissions scenario

In order to implement a net zero investing policy, asset managers and owners have to define a net zero scenario, which is summarized by a decarbonization pathway.

3.1 Paris-aligned benchmark pathways

To implement the Paris agreement on climate change, the European Union has created two climate benchmark labels: climate transition benchmark (CTB) and Paris-aligned benchmark (PAB). These two labels are structured along the following common principles:

1. A year-on-year self-decarbonization $\Delta\boldsymbol{\mathcal{R}}$ on average per annum, based on scope 1, 2 and 3 emissions intensity;
2. A minimum carbon intensity reduction $\boldsymbol{\mathcal{R}}^-$ compared to the investable universe;
3. A minimum exposure to sectors highly exposed to climate change;
4. A set of exclusion rules.

We deduce that the decarbonization pathway is defined by:

$$\boldsymbol{\mathcal{R}}(t_0, t) = 1 - (1 - \Delta\boldsymbol{\mathcal{R}})^{t-t_0} \left(1 - \boldsymbol{\mathcal{R}}^-\right) \tag{1}$$

where t_0 is the base year, t is the year index, and $\boldsymbol{\mathcal{R}}(t_0, t)$ is the reduction rate of the carbon footprint between t_0 and t. For the CTB label, the minimum reduction $\boldsymbol{\mathcal{R}}^-$ is set to 30% whereas it is equal to 50% for the PAB label. Moreover, the additional reduction rate $\Delta\boldsymbol{\mathcal{R}}$ is set to 7% for the two labels. Formula (1) can be used to create other decarbonization pathways. For instance, Figure 1 compares several trajectories of $\boldsymbol{\mathcal{R}}(t_0, t)$ by assuming that the base year is 2020. We notice that if $\Delta\boldsymbol{\mathcal{R}}$ is sufficiently large, the choice of the initial reduction rate $\boldsymbol{\mathcal{R}}^-$ has little impact on the long-run reduction rate $\boldsymbol{\mathcal{R}}(2020, 2050)$.

3.2 Carbon budget constraint

While CTB and PAB are the most known pathways in finance, their construction lacks theoretical and solid foundations. Indeed, they have been created *ex nihilo* such that the carbon footprint is close to zero by 2050, but they have no physical or economic foundations.

In fact, a net zero emissions (NZE) scenario corresponds to a carbon pathway, which is compatible with a carbon budget:

> *Using global mean surface air temperature, as in AR5, gives an estimate of the remaining carbon budget of 580 $GtCO_2e$ for a 50% probability of limiting warming to* 1.5°C*, and 420 $GtCO_2e$ for a 66% probability* (IPCC, 2018, page 26).

Let $\boldsymbol{\mathcal{CE}}(t)$ be the global carbon emissions at time t and $\boldsymbol{\mathcal{CB}}(t_0, t)$ be the global carbon budget between t_0 and t (Le Guenedal *et al.*, 2022):

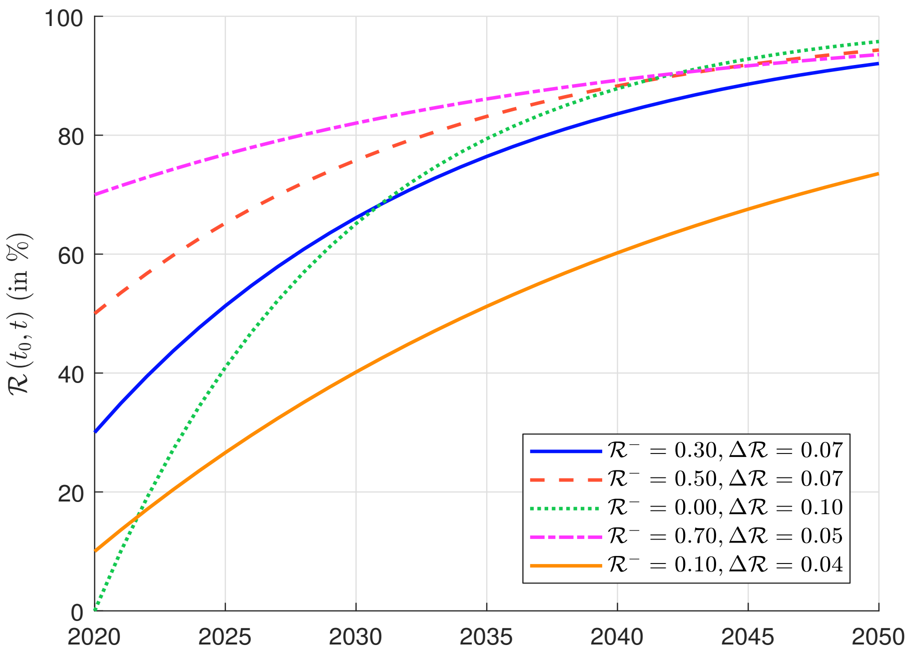
$$\boldsymbol{\mathcal{CB}}(t_0, t) = \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}(s)\, \mathrm{d}s \tag{2}$$

A NZE scenario can be defined by a carbon pathway that satisfies the following constraints:

$$\begin{cases} \boldsymbol{\mathcal{CB}}(t_0, 2050) \leq \boldsymbol{\mathcal{CB}}^+ \ \mathrm{GtCO_2e} \\ \boldsymbol{\mathcal{CE}}(2050) \approx 0 \ \mathrm{GtCO_2e} \end{cases} \tag{3}$$

where t_0 is the base date and $\boldsymbol{\mathcal{CB}}^+$ is the maximum carbon budget.

Figure 1: Examples of decarbonization pathway (in %)



Remark 2. *If we consider the AR5 results of IPCC (2018), we can set* $t_0 = 2019$ *and* $\boldsymbol{\mathcal{CB}}^+ = 580$. *If we would like to increase the probability that the global warming remains under* $1.5°$C*, the maximum carbon budget* $\boldsymbol{\mathcal{CB}}^+$ *can be replaced by a lower figure. Over the years, the budget constraint is moving, especially if the decarbonization pathway of the economy is not satisfied. For instance, the previous constraint* $\boldsymbol{\mathcal{CB}}(2019, 2050) \leq 580$ *GtCO*$_2$*e is generally updated and has become* $\boldsymbol{\mathcal{CB}}(2021, 2050) \leq 500$ *GtCO*$_2$*e.*

If we consider the decarbonization pathway given in Equation (1), we have:

$$\begin{aligned} \boldsymbol{\mathcal{CE}}(t) &= \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \boldsymbol{\mathcal{CE}}(t_0) \\ &= (1 - \Delta\boldsymbol{\mathcal{R}})^{t - t_0} \left(1 - \boldsymbol{\mathcal{R}}^-\right) \boldsymbol{\mathcal{CE}}(t_0) \end{aligned} \tag{4}$$

Using the analytical expression given in Le Guenedal *et al.* (2022, Equation (105), page 56), we obtain:

$$\boldsymbol{\mathcal{CB}}(t_0, t) = \left(\frac{(1 - \Delta\boldsymbol{\mathcal{R}})^{t - t_0} - 1}{\ln(1 - \Delta\boldsymbol{\mathcal{R}})}\right) \left(1 - \boldsymbol{\mathcal{R}}^-\right) \boldsymbol{\mathcal{CE}}(t_0) \tag{5}$$

By considering several values of $\boldsymbol{\mathcal{R}}^-$ and $\Delta\boldsymbol{\mathcal{R}}$, and assuming that $\boldsymbol{\mathcal{CE}}(2020) = 36$ GtCO$_2$e we obtain the figures given in Table 1. For instance, the carbon budget $\boldsymbol{\mathcal{CB}}(2020, 2050)$ is equal to 308 GtCO$_2$e if $\boldsymbol{\mathcal{R}}^- = 30\%$ and $\Delta\boldsymbol{\mathcal{R}} = 7\%$.

3.3 The IEA scenario

We must be careful with the specification of a decarbonization pathway, because its interpretation may differ from one application to another. Indeed, a decarbonization pathway is generally valid for an economy or a country. In this case, it is defined with respect to absolute carbon emissions. However, portfolio decarbonization uses carbon intensity, and not carbon emissions.

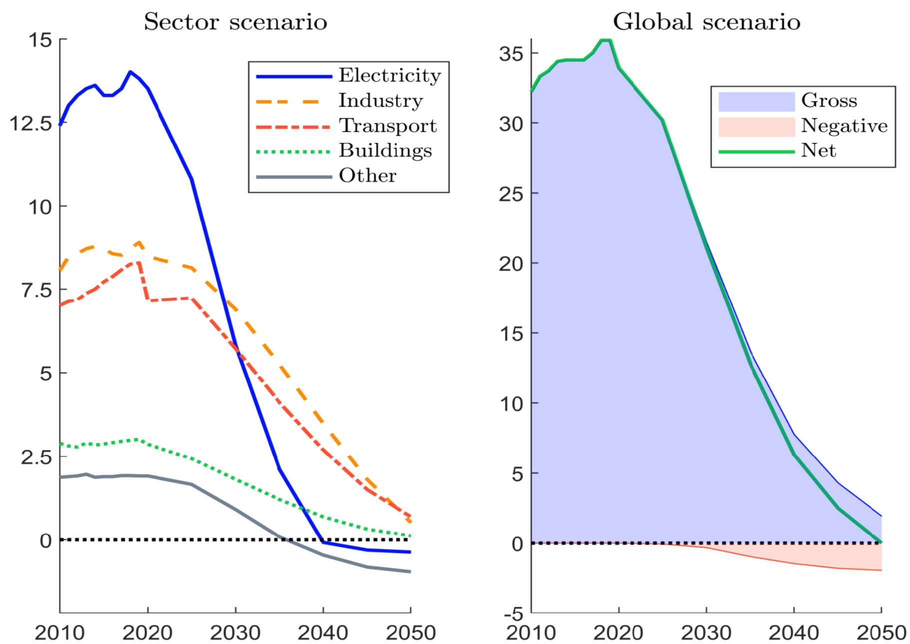
Let us consider the International Energy Agency (IEA) net zero scenario (IEA, 2021). IEA has analyzed each important sector to list the existing technologies and the future innovations that can

Table 1: Carbon budget $\mathcal{CB}(2020, 2050)$ of decarbonization pathways (in $GtCO_2e$)

	$\mathcal{R}^-$	0%	10%	20%	30%	50%	75%
$\Delta\mathcal{R}$	5%	551	496	441	386	276	138
	6%	491	442	393	344	245	123
	7%	440	396	352	308	220	110
	8%	396	357	317	277	198	99
	9%	359	323	287	251	180	90
	10%	327	294	262	229	164	82

help to reach net zero by 2050. For each sector, they have computed the resulting decarbonization pathway represented in the first panel in Figure 2. We notice that the power generation sector is the main contributor followed by the industry and transport sectors. The global decarbonization pathway[3] can then be deduced by summing all the sector trajectories and is reported in the second panel in Figure 2. We observe an acceleration of the decarbonization rate after 2025.

Figure 2: CO_2 emissions by sector in the IEA NZE scenario (in $GtCO_2e$)



Source: IEA (2021).

To compute the carbon budget $\mathcal{CB}(2019, 2050)$, we consider that the carbon pathway is a piecewise linear function. Therefore, we assume that $\mathcal{CE}(s)$ is known for $s \in \{t_0, t_1, \ldots, t_m = t\}$

[3]The IEA scenario gross CO_2 emissions in $GtCO_2e$ are equal to:

Year	2019	2020	2025	2030	2035	2040	2045	2050
$\mathcal{CE}(t)$	35.90	33.90	30.30	21.50	13.70	7.77	4.30	1.94

These figures are used to calibrate several pathways in the sequel.

and $\mathcal{CE}(s)$ is linear between two consecutive dates:

$$\mathcal{CE}(s) = \mathcal{CE}(t_{k-1}) + \frac{\mathcal{CE}_i(t_k) - \mathcal{CE}_i(t_{k-1})}{t_k - t_{k-1}}(s - t_{k-1}) \qquad \text{if } s \in [t_{k-1}, t_k] \tag{6}$$

Le Guenedal *et al.* (2022, Equation (112), page 57) has demonstrated that:

$$\begin{aligned}\mathcal{CB}(t_0, t) &= \frac{1}{2}\sum_{k=1}^{m}\left(\mathcal{CE}(t_k) - \mathcal{CE}(t_{k-1})\right)(t_k + t_{k-1}) + \\ &\quad \sum_{k=1}^{m}\left(\mathcal{CE}_i(t_{k-1})\, t_k - \mathcal{CE}(t_k)\, t_{k-1}\right)\end{aligned} \tag{7}$$

Using the IEA scenario, we obtain $\mathcal{CB}(2019, 2050) = 512.35$ $\mathrm{GtCO_2e}$. Since the two equations of the system (3) are satisfied[4], the IEA scenario can be considered as a 2050 net zero emissions scenario.

3.4 Relationships between carbon intensity and carbon emissions pathways

3.4.1 Relationship between reduction rates

Analytical method By definition, the carbon intensity $\mathcal{CI}(t)$ is defined as the ratio between the carbon emissions $\mathcal{CE}(t)$ and the normalization variable $Y(t)$:

$$\mathcal{CI}(t) = \frac{\mathcal{CE}(t)}{Y(t)} \tag{8}$$

Let $\mathcal{R}_{\mathcal{CI}}(t_0, t)$ and $\mathcal{R}_{\mathcal{CE}}(t_0, t)$ be the reduction rates of carbon intensity and emissions between t_0 and t. We have the following relationship:

$$\begin{aligned}\mathcal{R}_{\mathcal{CI}}(t_0, t) &= \frac{\mathcal{CI}(t_0) - \mathcal{CI}(t)}{\mathcal{CI}(t_0)} \\ &= \frac{g_Y(t_0, t) + \mathcal{R}_{\mathcal{CE}}(t_0, t)}{1 + g_Y(t_0, t)}\end{aligned} \tag{9}$$

where $g_Y(t_0, t)$ is the growth rate of the normalization variable. Generally, we assume that[5] $g_Y(t_0, t) \geq 0$ and $0 \leq \mathcal{R}_{\mathcal{CE}}(t_0, t) \leq 1$. Therefore, we can show the following property:

$$\left\{\begin{array}{l} g_Y(t_0, t) \geq 0 \\ 0 \leq \mathcal{R}_{\mathcal{CE}}(t_0, t) \leq 1 \end{array}\right. \Rightarrow \mathcal{R}_{\mathcal{CI}}(t_0, t) \geq \mathcal{R}_{\mathcal{CE}}(t_0, t) \tag{10}$$

We conclude that the reduction rate of the carbon intensity is always greater than the reduction rate of the carbon emissions.

Remark 3. *The emissions and intensity decarbonization pathways $\mathcal{R}_{\mathcal{CE}}(t_0, t)$ and $\mathcal{R}_{\mathcal{CI}}(t_0, t)$ are also called the 'economic' and 'financial' decarbonization pathways.*

Most of the time, we consider that the annual growth rate of the normalization variable is constant: $Y(t) = (1 + g_Y)\, Y(t-1)$. We deduce that the compound growth rate is equal to:

$$g_Y(t_0, t) = (1 + g_Y)^{t - t_0} - 1 \tag{11}$$

[4] We assume that $\mathcal{CB}^+ = 580$.

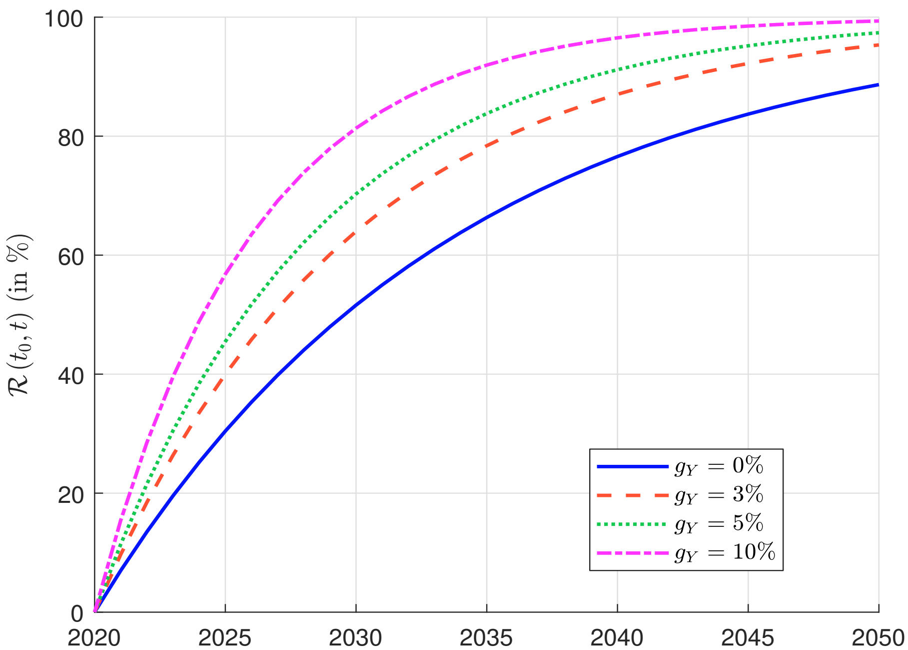
[5] For example, we anticipate that the sales or the revenues are increasing over time.

If we also assume that the annual reduction rate of carbon emissions is constant – $\mathcal{CE}(t) = (1 - \mathcal{R}_{\mathcal{CE}}) \mathcal{CE}(t-1)$, we obtain $\mathcal{R}_{\mathcal{CE}}(t_0, t) = 1 - (1 - \mathcal{R}_{\mathcal{CE}})^{t-t_0}$ and:

$$\mathcal{R}_{\mathcal{CI}}(t_0, t) = 1 - \left(1 - \frac{(g_Y + \mathcal{R}_{\mathcal{CE}})}{1 + g_Y}\right)^{t-t_0} \tag{12}$$

Equation (12) is the mirror formula of Equation (9) in the case of constant rates. Therefore, the annualized reduction rate of carbon intensity is approximatively equal to $g_Y + \mathcal{R}_{\mathcal{CE}}$. This implies that the intensity decarbonization pathway must be more aggressive than the emissions decarbonization pathway, as illustrated in Figure 3.

Figure 3: Impact of the growth rate g_Y on the intensity decarbonization pathway (in %) — $\mathcal{R}_{\mathcal{CE}}$ is set to 7%



Estimation method Let us consider a given economic decarbonization pathway $\{\mathcal{R}_{\mathcal{CE}}(t_0, t), t = t_1, \ldots, t_m\}$ and a given trajectory of the normalization variable growth $\{g_Y(t_0, t), t = t_1, \ldots, t_m\}$, we can use Equation (9) to compute the resulting financial decarbonization pathway $\{\mathcal{R}_{\mathcal{CI}}(t_0, t), t = t_1, \ldots, t_m\}$. If we assume that the functional form of the carbon intensity reduction is equal to:

$$f_1\left(t; \mathcal{R}^-_{\mathcal{CI}}, \Delta\mathcal{R}_{\mathcal{CI}}\right) = 1 - (1 - \Delta\mathcal{R}_{\mathcal{CI}})^{t-t_0}\left(1 - \mathcal{R}^-_{\mathcal{CI}}\right) \tag{13}$$

we can postulate the following regression model:

$$\mathcal{R}_{\mathcal{CI}}(t_0, t) = f_1\left(t; \mathcal{R}^-_{\mathcal{CI}}, \Delta\mathcal{R}_{\mathcal{CI}}\right) + \varepsilon(t) \tag{14}$$

and estimate the parameters $\left(\mathcal{R}^-_{\mathcal{CI}}, \Delta\mathcal{R}_{\mathcal{CI}}\right)$ by least squares.

By using the IEA net zero emissions scenario and considering linear interpolation scheme[6], we compute the emissions decarbonization pathway $\mathcal{R}_{\mathcal{CE}}(t_0, t)$ between 2020 and 2050 in Table 2. We

[6]We assume that the current carbon emissions $\mathcal{CE}(2020)$ are equal to 36 $GtCO_2e$.

Table 2: Intensity decarbonization pathways (in %) deduced from the IEA net zero emissions scenario

t	$\mathcal{R}_{\mathcal{CE}}(t_0,t)$	$\mathcal{R}_{\mathcal{CI}}(t_0,t)$ $g_Y=3\%$	$g_Y=5\%$	$g_Y=10\%$	$g_Y=20\%$	EU labels CTB	PAB
2020	0.0	0.0	0.0	0.0	0.0	30.0	50.0
2021	3.2	6.0	7.8	12.0	19.3	34.9	53.5
2022	6.3	11.7	15.0	22.6	35.0	39.5	56.8
2023	9.5	17.2	21.8	32.0	47.6	43.7	59.8
2024	12.7	22.4	28.2	40.4	57.9	47.6	62.6
2025	15.8	27.4	33.1	47.7	66.2	51.3	65.2
2026	20.7	33.6	40.8	55.2	73.5	54.7	67.7
2027	25.6	39.5	47.1	61.8	79.2	57.9	69.9
2028	30.5	45.1	52.0	67.6	83.8	60.8	72.0
2029	35.4	50.5	58.4	72.6	87.5	63.6	74.0
2030	40.3	55.6	63.3	77.0	90.4	66.1	75.8
2035	61.9	75.6	81.7	90.9	97.5	76.4	83.2
2040	78.4	88.0	91.9	96.8	99.4	83.6	88.3
2045	88.1	94.3	96.5	98.9	99.9	88.6	91.9
2050	94.6	97.8	98.8	99.7	100.0	92.1	94.3
$\mathcal{R}^{-}_{\mathcal{CI}}$	-12.6	-8.7	-6.8	-3.7	-1.3	30.0	50.0
$\Delta\mathcal{R}_{\mathcal{CI}}$	7.1	9.2	10.6	13.9	20.3	7.0	7.0

also deduce the intensity decarbonization pathway $\mathcal{R}_{\mathcal{CI}}(t_0,t)$ for different values of the constant growth rate g_Y. The comparison with CTB and PAB labels clearly shows that these last ones are very aggressive pathways for the next ten years. For instance, if we consider that $g_Y=5\%$, $\mathcal{R}_{\mathcal{CI}}(2020,2025)$ is equal to 33.1% for the IEA NZE scenario, whereas this figure is equal to 51.3% and 65.2% for CTB and PAB labels. In Table 2, we have also reported the estimated values[7] $\mathcal{R}^{-}_{\mathcal{CI}}$ and $\Delta\mathcal{R}_{\mathcal{CI}}$.

3.4.2 The carbon budget approach

Since we have $\mathcal{CE}(t)=Y(t)\,\mathcal{CI}(t)$, we obtain:

$$\begin{aligned}
\mathcal{CB}(t_0,t) &= \mathcal{CE}(t_0)\int_{t_0}^{t}\left(1+g_Y(t_0,s)\right)\left(1-\mathcal{R}_{\mathcal{CI}}(t_0,s)\right)\,\mathrm{d}s \\
&= \underbrace{(t-t_0)\,\mathcal{CE}(t_0)}_{\mathcal{CB}_1(t_0,t)}+\underbrace{\mathcal{CE}(t_0)\int_{t_0}^{t}g_Y(t_0,s)\,\mathrm{d}s}_{\mathcal{CB}_2(t_0,t)}- \\
&\qquad \underbrace{\mathcal{CE}(t_0)\int_{t_0}^{t}\left(1+g_Y(t_0,s)\right)\mathcal{R}_{\mathcal{CI}}(t_0,s)\,\mathrm{d}s}_{\mathcal{CB}_3(t_0,t)}
\end{aligned} \tag{15}$$

We can break-down the carbon budget into three components. The first component $\mathcal{CB}_1(t_0,t)$ corresponds to the total carbon emissions if nothing is done[8]. The second component $\mathcal{CB}_2(t_0,t)$

[7] We use a yearly partition between 2020 and 2050.

[8] This means that the emitted carbon emissions are stable.

corresponds to the extra carbon budget if the carbon intensity remains unchanged[9]. The third component $\mathcal{CB}_3(t_0,t)$ is the removed carbon budget due to the intensity reduction.

Let us assume that the annual growth rate of $Y(t)$ is constant and we use the PAB/CTB formula for the intensity decarbonization pathway. We deduce that:

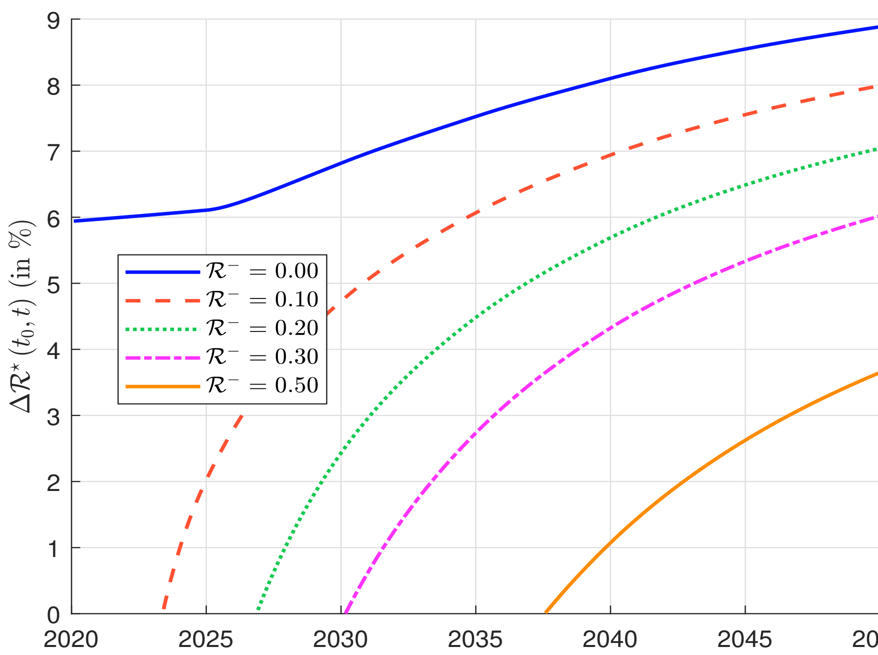
$$\begin{aligned}\mathcal{CB}(t_0,t) &= \frac{(1+g_Y)^{t-t_0}(1-\Delta\mathcal{R}_{\mathcal{CI}})^{t-t_0}-1}{\ln(1+g_Y)+\ln(1-\Delta\mathcal{R}_{\mathcal{CI}})}\left(1-\mathcal{R}_{\mathcal{CI}}^{-}\right)\mathcal{CE}(t_0) \\ &= f_2\left(t;\mathcal{R}_{\mathcal{CI}}^{-},\Delta\mathcal{R}_{\mathcal{CI}},g_Y\right) \qquad (16)\end{aligned}$$

If we consider a given carbon budget $\mathcal{CB}(t_0,t)$ and we assume a value for the growth rate g_Y, it is possible to estimate the parameters $\mathcal{R}_{\mathcal{CI}}^{-}$ and $\Delta\mathcal{R}_{\mathcal{CI}}$ by using the least squares approach. Another method consists in fixing the initial reduction rate $\mathcal{R}_{\mathcal{CI}}^{-}$ and to find the optimal value $\Delta\mathcal{R}_{\mathcal{CI}}$ such that the carbon budget is satisfied[10]:

$$\Delta\mathcal{R}^{\star}(t_0,t)=\inf\left\{\theta : f_2\left(t;\mathcal{R}_{\mathcal{CI}}^{-},\theta,g_Y\right)\le\mathcal{CB}(t_0,t)\right\}$$

By construction, $\Delta\mathcal{R}^{\star}(t_0,t)$ depends on the time horizon t because it is valid for the period $[t_0,t]$.

Figure 4: Estimated value $\Delta\mathcal{R}^{\star}(2020,t)$ (in %) from the IEA NZE scenario — $g_Y=3\%$



We use the IEA NZE scenario and estimate $\Delta\mathcal{R}_{\mathcal{CI}}^{\star}(2020,t)$ for several values of $\mathcal{R}_{\mathcal{CI}}^{-}$ when the gross rate g_Y is set equal to 3%. Results are reported in Figure 4. When $\mathcal{R}_{\mathcal{CI}}^{-}$ is equal to zero,

[9]If the carbon intensity is constant, this implies that the carbon budget increases and we have $\mathcal{CB}(t_0,t)=\mathcal{CB}_1(t_0,t)+\mathcal{CB}_2(t_0,t)$.

[10]This is equivalent to solve this non-linear inequation:

$$\frac{(1-\Delta\mathcal{R}_{\mathcal{CI}})^{t-t_0}-1}{\ln(1+g_Y)+\ln(1-\Delta\mathcal{R}_{\mathcal{CI}})}\le\frac{\mathcal{CB}(t_0,t)}{(1+g_Y)^{t-t_0}\left(1-\mathcal{R}_{\mathcal{CI}}^{-}\right)\mathcal{CE}(t_0)} \qquad (17)$$

the optimal reduction rate is close to 6% if the time horizon is short (less than 2025), whereas it reaches 9% if the time horizon is 2050. If the investor uses an initial reduction rate ($\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}^{-} > 0$), the additional reduction rate is implemented later. For instance, it is implemented after 2030 if $\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}^{-} = 30\%$. These results illustrate the aggressive behavior of the PAB pathway compared to the IEA pathway since the decarbonization velocity will increase only in the last 12 years with an additional rate lower than 4%.

In the previous approach, the optimal decarbonization rate $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0, t)$ could be viewed as the average value of $\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}$ that must be implemented between t_0 and t. It does not give the reduction rate we must consider after the time horizon. This is why we consider a third calibration approach, whose goal is to estimate the instantaneous decarbonization rate that must be implemented at time t. For that, we use the Chasles decomposition:

$$\boldsymbol{\mathcal{CB}}(t_0, t+h) = \boldsymbol{\mathcal{CB}}(t_0, t) + \int_t^{t+h} \boldsymbol{\mathcal{CE}}(s)\ \mathrm{d}s \tag{18}$$

where:

$$\begin{aligned}
\int_t^{t+h} \boldsymbol{\mathcal{CE}}(s)\ \mathrm{d}s &= \left(1 - \boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}^{-}\right) \boldsymbol{\mathcal{CE}}(t_0) \int_t^{t+h} (1+g_Y)^{s-t_0} (1 - \Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}})^{s-t_0}\ \mathrm{d}s \\
&= \frac{x^{t-t_0}\left(x^h - 1\right)}{\ln x} \left(1 - \boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}^{-}\right) \boldsymbol{\mathcal{CE}}(t_0) \\
&= f_3\left(t, h; \boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}^{-}, \Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}, g_Y\right)
\end{aligned} \tag{19}$$

and:

$$x = (1+g_Y)(1 - \Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}) \tag{20}$$

Therefore, the instantaneous decarbonization rate is the optimal value $\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}$ that satisfies the following equation:

$$\boldsymbol{\mathcal{R}}^{\star}(t) = \lim_{h \to 0} \inf \left\{\theta : \boldsymbol{\mathcal{CB}}(t_0, t) + f_3\left(t, h; \boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}^{-}, \theta, g_Y\right) \leq \boldsymbol{\mathcal{CB}}(t_0, t+h)\right\} \tag{21}$$

By construction, $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0, t)$ and $\boldsymbol{\mathcal{R}}^{\star}(t)$ may differ substantially. Indeed, we have:

$$1 - \boldsymbol{\mathcal{R}}(t_0, t) = \left(1 - \Delta\boldsymbol{\mathcal{R}}^{\star}(t_0, t)\right)^{t-t_0} \left(1 - \boldsymbol{\mathcal{R}}^{-}\right) \tag{22}$$

and:

$$\begin{aligned}
1 - \boldsymbol{\mathcal{R}}(t_0, t+h) &= \left(1 - \Delta\boldsymbol{\mathcal{R}}^{\star}(t_0, t+h)\right)^{t+h-t_0} \left(1 - \boldsymbol{\mathcal{R}}^{-}\right) \\
&\approx \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \left(1 - \boldsymbol{\mathcal{R}}^{\star}(t)\right)^h
\end{aligned} \tag{23}$$

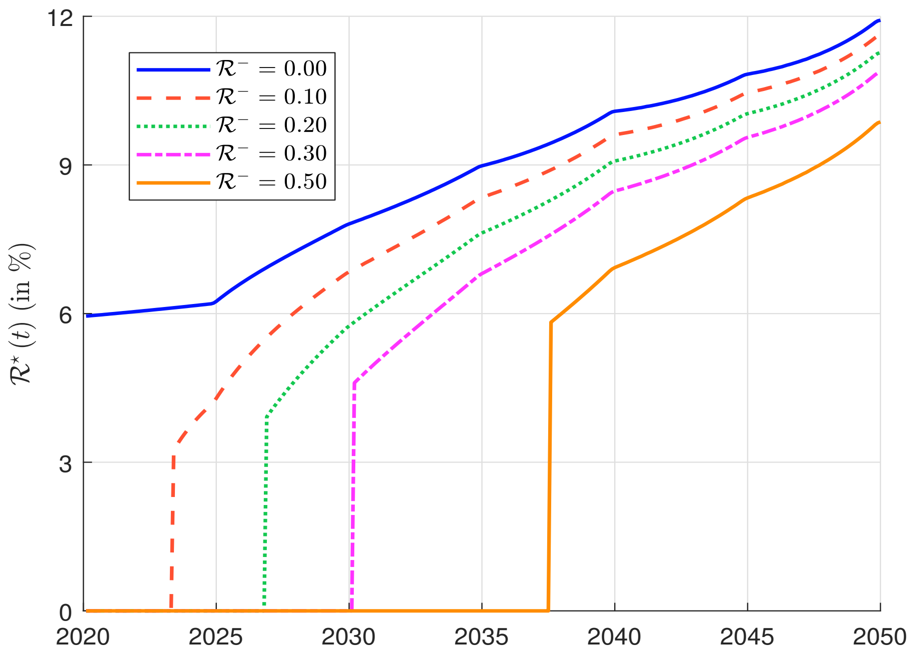
We deduce that:

$$\begin{aligned}
1 - \boldsymbol{\mathcal{R}}(t_0, t+\mathrm{d}t) &= \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \left(1 + \ln\left(1 - \boldsymbol{\mathcal{R}}^{\star}(t)\right)\ \mathrm{d}t\right) \\
&\approx \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \left(1 - \boldsymbol{\mathcal{R}}^{\star}(t)\ \mathrm{d}t\right) \\
&= \left(1 - \Delta\boldsymbol{\mathcal{R}}^{\star}(t_0, t)\right)^{t-t_0} \left(1 - \boldsymbol{\mathcal{R}}^{-}\right) \left(1 - \boldsymbol{\mathcal{R}}^{\star}(t)\ \mathrm{d}t\right)
\end{aligned} \tag{24}$$

In the case $\boldsymbol{\mathcal{R}}^{-} = 0$, we have the following approximation:

$$\begin{aligned}
\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0, t) &\approx -\frac{1}{t-t_0} \int_{t_0}^{t} \ln\left(1 - \boldsymbol{\mathcal{R}}^{\star}(s)\right)\ \mathrm{d}s \\
&\approx \frac{1}{t-t_0} \int_{t_0}^{t} \boldsymbol{\mathcal{R}}^{\star}(s)\ \mathrm{d}s
\end{aligned} \tag{25}$$

Figure 5: Estimated value $\mathcal{R}^{\star}(t)$ (in %) from the IEA NZE scenario — $g_Y = 3\%$



More generally, $\Delta\mathcal{R}^{\star}(t_0, t)$ can be viewed as an averaging function of $\mathcal{R}^{\star}(t)$. If $\Delta\mathcal{R}^{\star}(t_0, t)$ is an increasing function of t, we then expect that $\mathcal{R}^{\star}(t) > \Delta\mathcal{R}^{\star}(t_0, t)$.

In Figure 5, we have reported the instantaneous rate $\mathcal{R}^{\star}(t)$ for several values of $\mathcal{R}^{-}_{\mathcal{CI}}$. If we compare these plots with those given in Figure 4, we verify that $\mathcal{R}^{\star}(t) > \Delta\mathcal{R}^{\star}(t_0, t)$. Let us consider the case $\mathcal{R}^{-}_{\mathcal{CI}} = 0$. If the fund manager would like to follow the IEA NZE scenario and if we assume that $g_Y = 3\%$, he must decarbonize his portfolio with a rate of 6% at the beginning. Then, he must progressively increase the decarbonization rate to reach 12% by 2050.

Remark 4. *The previous instantaneous rate $\Delta\mathcal{R}^{\star}(t)$ is different from the classic definition*[11].

Remark 5. *In Appendix B on page 117, we compare the two decarbonization rates $\Delta\mathcal{R}^{\star}(t_0, t)$ and $\mathcal{R}^{\star}(t)$. We also report the logarithmic and arithmetic mean values. This confirms that $\Delta\mathcal{R}^{\star}(t_0, t)$ can be interpreted as the mean of $\mathcal{R}^{\star}(t)$.*

To illustrate the aggressive nature of CTB and PAB pathways, we first estimate the implied growth rate g_Y that fits the intensity reduction pathway. The least square estimates are respectively equal to $\hat{g}_Y = 6.70\%$ and $\hat{g}_Y = 16.27\%$ for CTB and PAB. However, the fitted pathway is not appealing (see Figure 38 on page 118). Another approach consists in matching the carbon budget: $g^{\star}_Y(t_0, t) = \sup\left\{\theta : f_2\left(t; \mathcal{R}^{-}_{\mathcal{CI}}, \Delta\mathcal{R}_{\mathcal{CI}}, \theta\right) \leq \mathcal{CB}(t_0, t)\right\}$. For instance, we obtain $g^{\star}_Y(2020, 2035) = 12.39\%$ for PAB. In Figure 39 on page 119, we have reported all the solutions $g^{\star}_Y(2020, t)$. These results clearly show that CTB and PAB pathways are too aggressive if we are confident in the IEA scenario.

[11] Since the relationship $\mathcal{CE}(t) = Y(t)\,\mathcal{CI}(t)$ can be written as $\ln\mathcal{CE}(t) = \ln Y(t) + \ln\mathcal{CI}(t)$, we deduce that $\mathrm{d}\ln\mathcal{CI}(t) = \mathrm{d}\ln\mathcal{CE}(t) - \mathrm{d}\ln Y(t)$. Let $\varrho_{\mathcal{CI}}(t)$ be the instantaneous rate of change. We have $\mathrm{d}\mathcal{CI}(t) = -\varrho_{\mathcal{CI}}(t)\,\mathcal{CI}(t)\,\mathrm{d}t$. This implies that $\varrho_{\mathcal{CI}}(t) = \ln(1 + g_Y) - \partial_t \ln\mathcal{CE}(t)$. We verify that $\Delta\mathcal{R}^{\star}(t) \neq \varrho_{\mathcal{CI}}(t)$.

4 Net zero metrics

Before investigating the construction of net zero portfolios, we have to define the metrics that are useful when implementating a net zero investment policy. As explained in the introduction, we must consider two dimensions: the decarbonization dimension and the transition dimension. Therefore, we consider two types of metrics. Net zero carbon metrics are used to assess the first dimension. They are generally related to the concept of carbon footprint. Net zero transition metrics are used to assess the second dimension. They measure the capacity for financing the transition to a low-carbon economy. Since net zero carbon metrics are generally physical measures expressed in CO_2e, net zero transition metrics are rather monetary measures expressed in dollars. Another important issue is the dynamic property of net zero investing. This is the big difference from a simple portfolio decarbonization exercise. Therefore, we must distinguish between static and dynamic (or forward-looking) measures. Indeed, a net zero emissions scenario is described by a trajectory. Net zero investing cannot be reduced to the process that locates the node of the trajectory corresponding to a given date. Net zero investing must imply a dynamic pathway that corresponds to the trajectory. This is the real challenge of net zero investing.

4.1 Net zero carbon metrics

4.1.1 Static measures of carbon footprint

Scope definition The GHG Protocol corporate standard classifies a company's greenhouse gas emissions in three scopes[12]:

- Scope 1 denotes direct GHG emissions occurring from sources that are owned and controlled by the issuer.
- Scope 2 corresponds to the indirect GHG emissions from the consumption of purchased electricity, heat or steam.
- Scope 3 are other indirect emissions (not included in scope 2) of the entire value chain. They can be divided into two main categories[13]:
  - Upstream scope 3 emissions[14] are defined as indirect carbon emissions related to purchased goods and services.
  - Downstream scope 3 emissions[15] are defined as indirect carbon emissions related to sold goods and services.

Scope 1 emissions are also called direct emissions, whereas indirect emissions encompass both scopes 2 and 3 GHG emissions. Unlike scopes 1 and 2, scope 3 is an optional reporting category. Moreover, indirect emissions may present big challenges in terms of double/triple counting. For

[12]The latest version of corporate accounting and reporting standard can be found at www.ghgprotocol.org/corporate-standard.

[13]The upstream value chain includes all activities related to the suppliers whereas the downstream value chain refers to post-manufacturing activities.

[14]In the GHG Protocol, the upstream scope 3 is based on 8 sub-categories: (1) purchased goods and services, (2) capital goods, (3) fuel and energy related activities, (4) upstream transportation and distribution, (5) waste generated in operations, (6) business travel, (7) employee commuting and (8) upstream leased assets.

[15]In the GHG Protocol, the downstream scope 3 is based on these next 7 sub-categories: (9) downstream transportation and distribution, (10) processing of sold products, (11) use of sold products, (12) end-of-life treatment of sold products, (13) downstream leased assets, (14) franchises and (15) investments.

instance, a large part of scope 2 may be found in scope 1 of Utilities companies that produce or distribute electricity. A part of upstream scope 3 is already present in Materials and Industrials companies, whereas another part of downstream scope 3 is embedded in Retailing and Distribution industries. Issues on data quality and double counting bias explain that portfolio decarbonization is generally based on scopes 1 and 2 emissions.

Data providers Data on GHG emissions are easily and freely available when they concern countries and regions[16]. For corporations, three main providers of GHG emissions are generally used: the CDP database, MSCI and the S&P Trucost data[17].

CDP (formerly known as the Carbon Disclosure Project) assists businesses and local governments in disclosing their environmental effect. By promoting disclosure, understanding, and action in the direction of a sustainable economy, it seeks to establish environmental reporting and risk management as business standards. Companies are asked to disclose annually their carbon emissions regarding the three scopes by completing the CDP questionnaire. Based on the data collected as of October 2021 in the CDP database, around 3 000 companies disclosed their 2020 carbon emissions and can be matched to a financial ISIN which is necessary to build portfolios. S&P Trucost then build on the CDP database and extents it to numerous companies thanks to a proprietary estimation model. Hence, Trucost data cover more than 15 000 companies. Finally MSCI Inc., a global provider of data, equity and fixed income indexes as well as multi-asset portfolio analysis tools, also provides carbon emissions data based on CDP reporting and its own statistical estimation. The latter covers the entire MSCI All World Country Investable market Index (ACWI IMI)[18].

A quick comparison of the three suppliers shows a slight fragility in the data. We sometimes face conversion errors where there are factors of 100, 1 000 or even 1 000 000 between the carbon emission of the same company provided by two different suppliers. While some errors are easy to spot and can be corrected, there are more insidious differences. To illustrate this, we place ourselves in the common universe of the three data providers. We then calculate the share of companies with similar carbon emissions data across the different suppliers at different thresholds. The results for scope 1 emissions can be found in table 3 below.

Table 3: Share (in %) of similar carbon emissions data between providers - $\boldsymbol{\mathcal{SC}}_1$

	Provider	Provider 1	Provider 2	Provider 3
Equality	Provider 1	100	27	42
	Provider 2	27	100	9
	Provider 3	42	9	100

	Provider	Provider 1	Provider 2	Provider 3
10% difference	Provider 1	100	62	68
	Provider 2	62	100	28
	P rovider 3	68	28	100

Scope 1 emissions data, although widely considered as strongly reliable, may lack of robustness over the data providers. Provider 2 and Provider 3 indeed only exactly match (up to 1 ton

[16]They can be retrieved from the World Bank (`data.worldbank.org/topic/climate-change`), Climate Watch Data (`www.climatewatchdata.org/ghg-emissions`), Global Carbon Project (`www.globalcarbonproject.org`), etc.

[17]A description of these two providers can be found at `www.cdp.net/en/data` and `www.spglobal.com/esg/trucost`.

[18]This index captures, as of June 2022, 9 187 companies from large, mid and small caps across developed and emerging markets.

difference) 27% and 42% of the Provider 1 data, even though it is labelled as disclosed by the company itself. Assuming the data are similar if we observe a relative difference lower than 10% between two providers, these figures become 62% for Provider 2 and 68% for Provider 3. Moreover, they only agree on scope 1 emissions data for 28% of the 3 000 considered companies. The same remark applies on both scope 2 and scope 3 carbon emissions and the related figures are shown in tables 40 and 41 in Appendix B.1.1. Breaking down scope 3 into upstream and downstream emissions in table 42 clearly shows the poor quality of upstream carbon emission estimates.

If we compute some quantiles of scope 3 carbon emissions, we observe that every provider depicts a different picture in table 4. Worse yet, we can find negative emissions and companies whose emissions are higher than the total global emissions as estimated by the IEA in 2019.

Table 4: Summary statistics of $\mathcal{SC}_3$ carbon emissions by provider

Provider	Provider 1	Provider 2	Provider 3
Min	0	-65	0
25%	19 832	34 171	11 515
50%	333 575	180 792	117 039
75%	3 842 134	927 295	2 258 764
Max	130 900 617 456	2 742 409 470	88 059 781 168

Finally, it is questionable whether there is a decline in quality induced by small capitalizations or developing markets, whose emissions are poorly reported, that explains these discrepancies. Table 5 and 6 focus on three of the 20 largest market capitalizations in the index. We find that there are two different figures for scope 2 emissions for company A provided by Providers 2 and 3. Provider 1 fails to give us any indication about the correct figure and reports the same two previous ones. Company B's scope 2 estimates appear to be more robust. We also note that even when considering scope 2 of large companies, some providers may not give any estimate at all. Moreover, the discrepancies tend to widen when considering scope 3 emissions, even for the largest capitalizations in the index. For example, we can notice a significant difference factor for scope 3 emissions of companies B and C.

Table 5: Comparison of 2019 $\mathcal{SC}_2$ carbon emissions across providers

Company	Provider 1 $\mathcal{SC}_2$ - reported 1	Provider 1 $\mathcal{SC}_2$ - reported 2	Provider 2 $\mathcal{SC}_2$	Provider 3 $\mathcal{SC}_2$
Company A	5 865 095	911 415	5 866 412	911 415
Company B	1 213 974	1 140 671	1 393 916	1 213 974
Company C			7 000 000	7 000 000

Table 6: Comparison of 2019 $\mathcal{SC}_3$ carbon emissions across providers

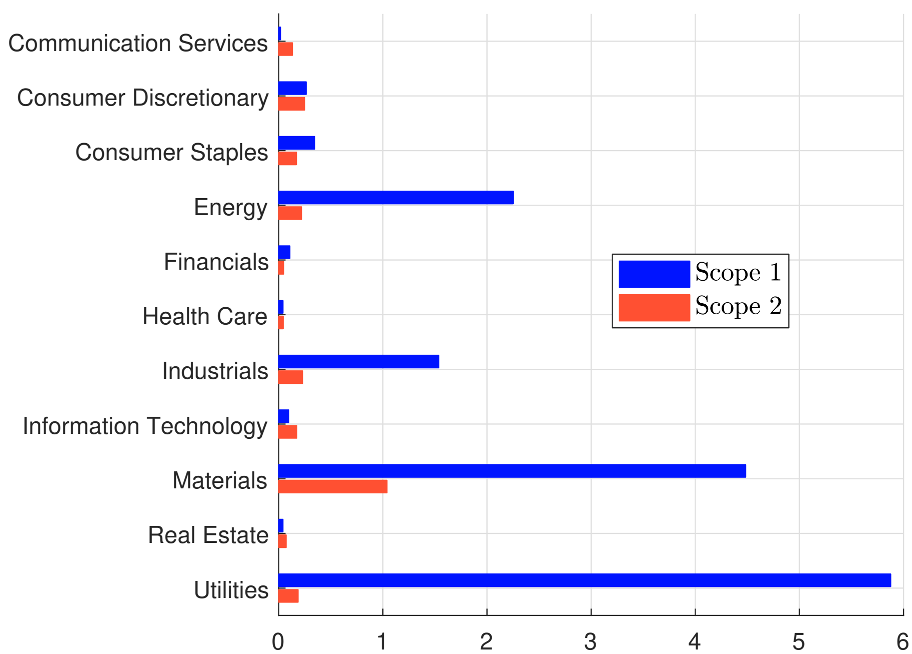
Company	Provider 1 $\mathcal{SC}_3$	Provider 2 $\mathcal{SC}_3$	Provider 3 $\mathcal{SC}_3$
Company A	9 376 000	8 090 698	9 376 000
Company B	185 746 651	7 611 372	185 746 651
Company C		85 038 490	540 000 000

Remark 6. *Further analysis could be conducted but our goal was here to raise awareness about the quality of carbon emissions data, not only regarding scope 3 emissions but at all levels. Through*

a sectoral breakdown analysis we find that the choice of provider has a significant impact on sector contribution to global carbon emissions. It is unfortunately delicate to deploy a systematized correction approach as it is obviously hard to guess which provider is the most accurate and it is not the aim of our work. As the considered universe is large and providers regularly update their estimates, it is difficult to analyse each company's emission and produce our own monitoring. This is, however, at the heart of the literature at the moment. Many works try to develop more accurate reporting rules alongside robust approach for carbon emissions estimation (Nguyen et al., 2020).

Carbon emissions S&P Trucost is undoubtedly the most widely used dataset among practitioners as it offers the best data coverage regarding every of the three carbon scopes. We therefore consider the Trucost dataset of carbon emissions as of 01/06/2022 and analyze the distribution of carbon emissions in 2019 for around 15 000 companies. We prefer to use the year 2019 instead of the year 2020, because the covid-19 crisis had a significant impact on the carbon footprint. In Figure 6, we have reported the scopes 1 and 2 carbon emissions per GICS sector. We notice that including scope 2 has a limited impact, except for some low-carbon sectors such as Consumer Services, Information Technology and Real Estate. In Table 43 on page 102, we have calculated the breakdown of carbon emissions. Scopes 1 and 2 represent 17.6 $GtCO_2e$, and the most important sectoral contributors are Utilities (34.4%), Materials (31.4%), Energy (14.0%) and Industrials (10.0%). This means that these 4 strategic sectors explain about 90% of scopes 1 and 2 carbon emissions.

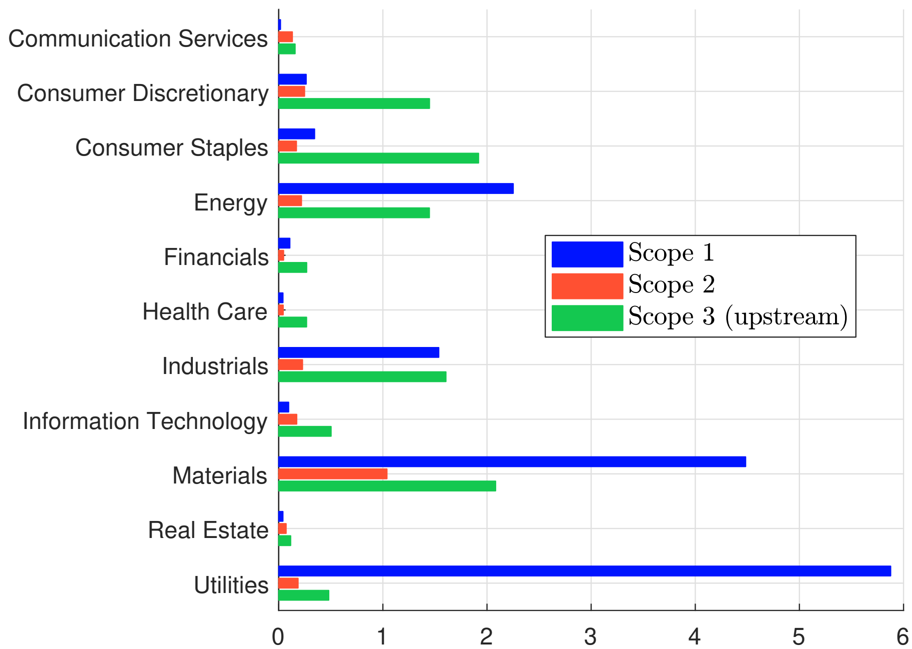
Figure 6: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1 & 2)



In Figure 7, we observe that some sectors are highly impacted by the upstream scope 3 emissions. For instance, the ratio $\dfrac{\mathcal{SC}_3^{\text{up}}}{\mathcal{SC}_{1-2}}$ is greater than 2.5 for Consumer Discretionary, Consumer Staples and Health Care, and is close to 2 for Information Technology[19]. Among the strategic

[19] See Table 44 on page 102.

Figure 7: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1, 2 & 3 upstream)



sectors, Energy and Industrials are the most penalized whereas the upstream scope 3 emissions of Utilities is relatively small compared to its scope 1 emissions.

While the impact of the upstream scope 3 is significant, the impact of the downstream scope 3 is huge as demonstrated in Figure 8. Four sectors have very large downstream carbon emissions: Consumer Discretionary, Energy, Industrials and Materials. While Utilities has the most important contribution in terms of scopes 1 and 2 since it represents 34.4% of carbon emissions, its contribution to scope 3 is relatively modest and is equal to 4.8%. Including or not scope 3, in particular the downstream carbon emissions, changes the whole picture of the breakdown between the sectors.

Remark 7. *When considering carbon emissions, double counting is a real issue. According to Table 43 on page 102, the total carbon emissions is* 17.6 *$GtCO_2e$ for scopes* $1 + 2$*, and* 81.6 *$GtCO_2e$ for scopes* $1 + 2 + 3$*, while we estimate that the world emits about* 36 *$GtCO_2e$ per year. This issue is discussed later.*

Figure 8: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1, 2, 3 upstream & 3 downstream)

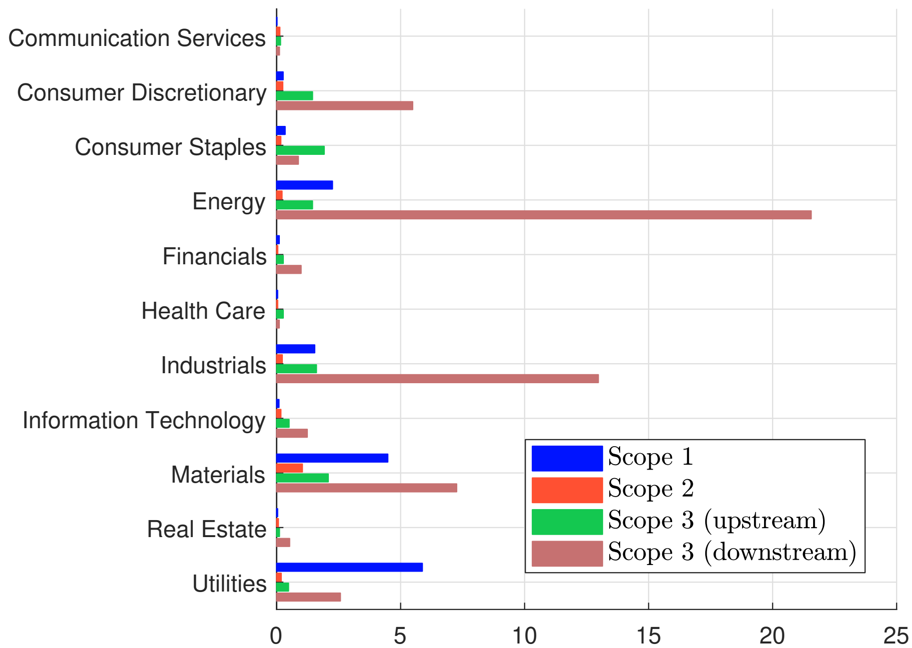
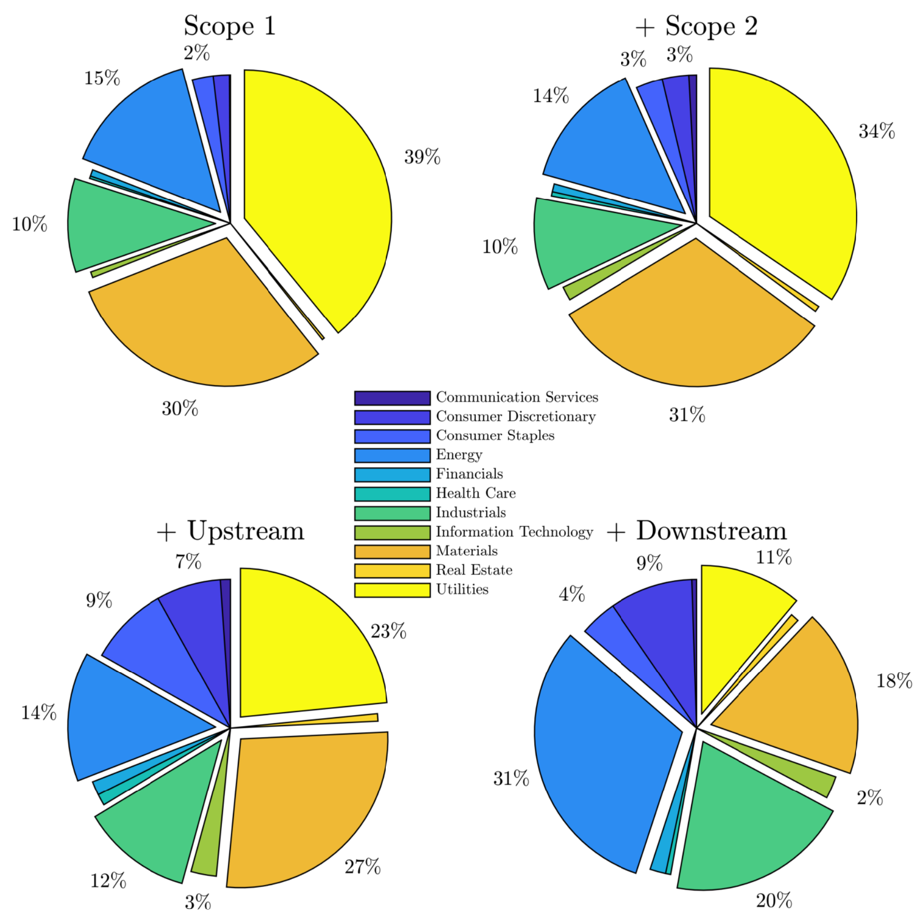


Figure 9 is a visualisation of the sectoral contribution by considering the addition of several scopes. At each step, the contribution of Materials and Utilities decreases whereas it increases for Consumer Discretionary, Energy, Industrials and Information Technology. Among the most significant sectors[20], the behavior of Consumer Staples is singular since its contribution increases when adding scope 2 and upstream scope 3, but decreases when considering downstream scope 3.

Figure 9: Sectoral carbon contribution (in %)



Carbon intensity From a financial point of view, it does not make sense to compare and aggregate the carbon emissions of a large cap company with the carbon emissions of a small cap company. Therefore, portfolio managers use the concept of carbon intensity, which is a normalization of the carbon emissions. The goal is then to compare and aggregate the carbon footprint of several issuers with different business sizes. From a mathematical point of view, we have:

$$\mathcal{CI} = \frac{\mathcal{CE}}{Y} \tag{26}$$

where $\mathcal{CE}$ is the company's carbon emissions and Y is an output indicator measuring its activity. We distinguish two categories: physical and monetary intensities. In the case of physical intensity, we generally use metrics that measure the production units[21]. In the case of monetary intensity,

[20] They correspond to sectors that have a contribution greater than 2%.

[21] For instance, we can express the carbon intensity in CO_2e/kWh for an Electricity company.

we can consider accounting or market-based metrics. For instance, we can use revenues or sales to normalize carbon emissions. Some examples are provided in Table 7. These figures illustrate some issues in the computation of the carbon footprint at the issuer level. First, it is obvious that it is important to take into account scope 3 to have the real picture of the carbon footprint of an issuer. Indeed, we notice that some issuers have a low scope 1, because they have more or less outsourced the manufacturing of their products. Since a part of the production is located in upstream scope 3, we can not make a fair comparison between issuers if we only consider scopes 1 and 2. We face a similar issue with the distribution of the products. This implies that a part of downstream scope 3 of some issuers may be located in scope 1 of other issuers.

The magnitude of some scope 3 carbon intensities raises the question of their computation. Indeed, while scopes 1 and 2 are mandatory to report, there is no obligation for a company to report its scope 3. Moreover, while there is one unique figure for scopes 1 and 2 in the CDP reporting files, scope 3 is split into 15 categories (See Footnotes 14 and 15 on page 29), and it is extremely rare that a company reports all scope 3 categories. This explains that the frequency of estimated values is larger for scope 3. How to compare the reported value for one company with the estimated value for another company? The answer is not obvious since the estimated value depends on the statistical model of the data provider. Moreover, it seems that the GHG protocol for scope 3 is not enough precise because we may observe very large differences between two reported companies of the same industry (GICS level 3).

In Figure 10, we show the distribution of carbon intensities. Since the range may be very large (from zero to several thousand), we use a logarithmic scale. Moreover, the dotted vertical lines indicate the 5^{th} and 95^{th} percentiles. We observe that the distribution support is very large for scopes 1, 2 and 3 downstream. In this case, there are many extreme points with very low and very high carbon intensities. Therefore, it is relatively easy to reduce the carbon footprint of a portfolio. We must remove corporates with the highest carbon intensity (for instance greater than 1 000) and replace them with corporates with the lowest carbon intensity (for instance less than 5). Now, if we focus on upstream scope 3, we obtain another story, because the range is not so large. Indeed, we do not have corporates with very low carbon intensity. Therefore, incorporating upstream scope 3 changes the nature of portfolio decarbonization.

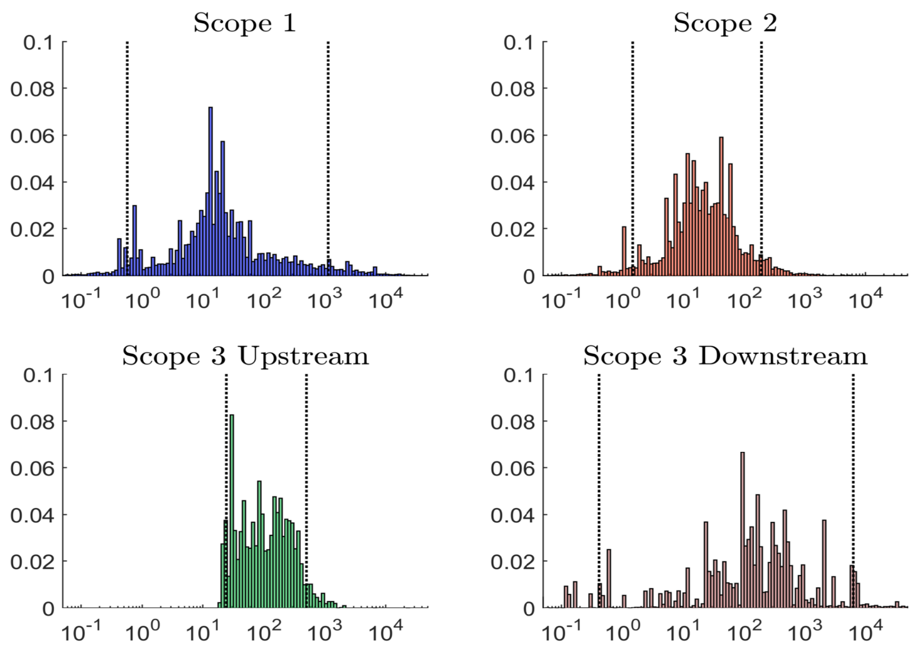
Remark 8. *The question of double-counting is less important when we consider carbon intensities, especially monetary measures. Indeed, the carbon intensity can be seen as a scoring system, and portfolio managers generally use carbon intensity in a relative way, and not in an absolute way. For instance, they do not target a given carbon intensity. Their goal is then to reduce the carbon intensity relatively to a benchmark, without analyzing the absolute value of the benchmark itself. Moreover, the aggregation at the portfolio level is generally done thanks to the WACI*[22] *measure, which indicates that the carbon intensity is more viewed as a score than a physical measure.*

[22] Weighted average of carbon intensity.

Table 7: Examples of 2019 carbon emissions and intensities

Company	Emission (in tCO_2e)				Revenue	Intensity (in tCO_2e/\$ mn)			
	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{\mathrm{up}}$	$\mathcal{SC}_3^{\mathrm{down}}$	(in \$ mn)	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{\mathrm{up}}$	$\mathcal{SC}_3^{\mathrm{down}}$
Airbus	576 705	386 674	12 284 183	23 661 432	78 899	7.3	4.9	155.7	299.9
Allianz	46 745	224 315	3 449 234	3 904 000	135 279	0.3	1.7	25.5	28.9
Amazon	5 760 000	5 500 000	20 054 722	10 438 551	280 522	20.5	19.6	71.5	37.2
Apple	50 549	862 127	27 624 282	5 470 771	260 174	0.2	3.3	106.2	21.0
BNP Paribas	64 829	280 789	1 923 307	1 884	78 244	0.8	3.6	24.6	0.0
Boeing	611 001	871 000	9 878 431	22 959 719	76 559	8.0	11.4	129.0	299.9
BP	49 199 999	5 200 000	103 840 194	582 639 687	276 850	177.7	18.8	375.1	2 104.5
Caterpillar	905 000	926 000	15 197 607	401 993 744	53 800	16.8	17.2	282.5	7 472.0
Danone	722 122	944 877	28 969 780	4 464 773	28 308	25.5	33.4	1 023.4	157.7
Enel	69 981 891	5 365 386	8 726 973	53 774 821	86 610	808.0	61.9	100.8	620.9
Exxon	111 000 000	9 000 000	107 282 831	594 131 943	255 583	434.3	35.2	419.8	2 324.6
JPMorgan Chase	81 655	692 299	3 101 582	15 448 469	115 627	0.7	6.0	26.8	133.6
LVMH	67 613	262 609	11 853 749	942 520	60 083	1.1	4.4	197.3	15.7
Microsoft	113 414	3 556 553	5 977 488	4 003 770	125 843	0.9	28.3	47.5	31.8
Nestle	3 291 303	3 206 495	61 262 078	33 900 606	93 153	35.3	34.4	657.6	363.9
PepsiCo	3 552 415	1 556 523	32 598 029	14 229 956	67 161	52.9	23.2	485.4	211.9
Pfizer	734 638	762 840	4 667 225	133 468	51 750	14.2	14.7	90.2	2.6
Roche	288 157	329 541	5 812 735	347 437	64 154	4.5	5.1	90.6	5.4
Samsung Electronics	5 067 000	10 998 000	33 554 245	60 978 947	197 733	25.6	55.6	169.7	308.4
TotalEnergies	40 909 135	3 596 127	49 817 293	456 993 576	200 316	204.2	18.0	248.7	2 280.0
Toyota	2 522 987	5 227 844	66 148 020	330 714 268	272 608	9.3	19.2	242.6	1 213.2
Volkswagen	4 494 066	5 973 894	65 335 372	354 913 446	282 817	15.9	21.1	231.0	1 254.9
Walmart	6 101 641	13 057 352	40 651 079	32 346 229	514 405	11.9	25.4	79.0	62.9

Figure 10: Distribution of carbon intensities (logarithmic scale)

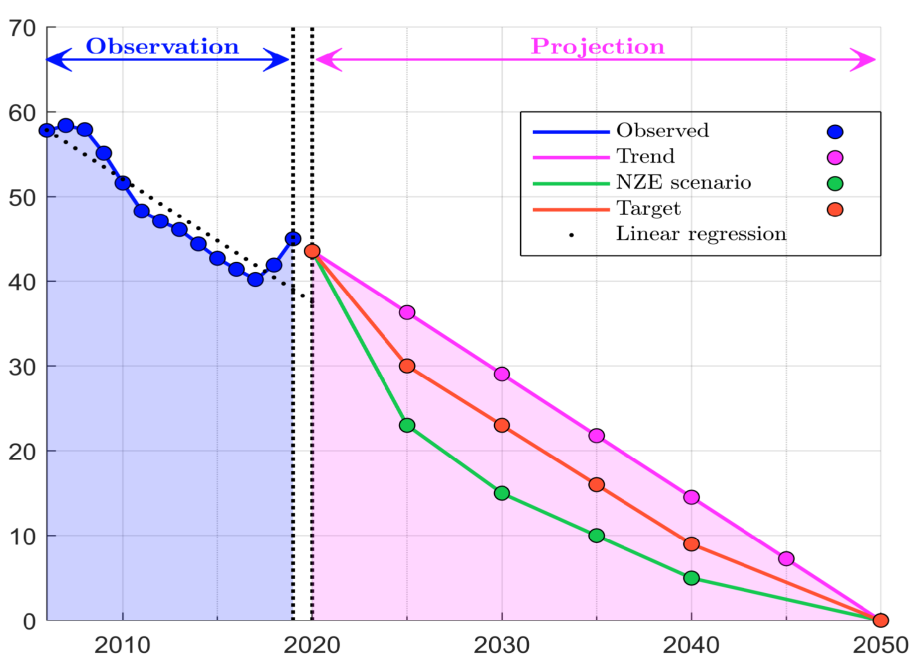


Source: Trucost (2022).

4.1.2 Dynamic measures of carbon footprint

The $\mathcal{PAC}$ framework Dynamic measures of carbon emissions (or net zero carbon metrics) are generally defined according to the $\mathcal{PAC}$ framework (Le Guenedal *et al.*, 2022). $\mathcal{PAC}$ stands for participation, ambition and credibility. Its purpose is to evaluate the decarbonization capacity and willingness of issuers. To understand this framework, we consider the example given in Figure 11. For a given issuer, we have reported the historical trajectory of carbon emissions from 2005 to 2019 (blue line). Therefore, we can estimate the associated linear trend model and project the future carbon emissions by assuming that the issuer will do the same efforts in the future than in the past (violet line). Therefore, the participation pillar measures the past efforts of the issuer. In our example, the carbon trend is negative, meaning that the issuer has globally reduced its carbon emissions in the past. Moreover, we notice that the issuer can reach net zero by 2050 if it continues its efforts. The participation of this issuer is then good and positive. The second pillar measures the ambition of the issuer, and compares the target trajectory on one side (red line) and the net zero scenario of the sector on the other side (green line). The underlying idea is to assess the announcements of the issuer concerning its net zero policy. In our case, the target trajectory being above the net zero scenario, this issuer has not a lot of ambition. Finally, we can measure the credibility of the targets by comparing the current trend of carbon emissions (violet line) and the reduction targets announced by the company (red line). In our case, the credibility of the issuer is good and positive. The $\mathcal{PAC}$ framework described above constitutes the backbone of temperature ratings provided by data providers.

Figure 11: Illustration of the $\mathcal{PAC}$ framework



Carbon momentum Temperature ratings may be viewed as black-box systems. This is why some portfolio managers prefer to focus on the participation pillar since it only depends on the historical trajectory. Le Guenedal *et al.* (2022) define the carbon trend by considering the linear constant trend model:

$$\mathcal{CE}(t) = \beta_0 + \beta_1 \cdot t + u(t) \tag{27}$$

Using the least squares method, we can estimate β_0 and β_1. Let t_0 be the base year. We can build the carbon trajectory implied by the current trend by applying the projection:

$$\widehat{\mathcal{CE}}(t) = \mathcal{CE}(t_0) + \hat{\beta}_1 \cdot (t - t_0) \tag{28}$$

for $t \geq t_0$. This model is very simple since the underlying idea is to extrapolate the past trajectory. Following Le Guenedal *et al.* (2022), we can consider a dynamic version of the estimation method and we note $\hat{\beta}_1(t)$ the slope coefficient of the trend model that is estimated at time t. We define the long-term carbon momentum as the ratio between the slope and the current carbon emissions:

$$\mathcal{CM}^{\mathcal{L}ong}(t) = \frac{\hat{\beta}_1(t)}{\mathcal{CE}(t)} \tag{29}$$

Le Guenedal *et al.* (2022) also introduce the concept of carbon velocity, which measures the normalized slope change between $t - h$ and t:

$$\boldsymbol{v}^{(h)}(t) = \frac{\hat{\beta}_1(t) - \hat{\beta}_1(t-h)}{h} \tag{30}$$

The rationale for this measure is the following. A net zero emissions commitment implies a negative trend: $\hat{\beta}_1(t) < 0$. Nevertheless, it can take many years for a company to change the sign of the trend slope if it has a bad track record. Therefore, we can use the velocity to verify that the company is making significant efforts in the recent period. In this case, we must have $\boldsymbol{v}^{(h)}(t) < 0$ for low values[23] of h. Therefore, the short-term carbon momentum is defined as:

$$\mathcal{CM}^{\mathcal{S}hort}(t) = \frac{\boldsymbol{v}^{(1)}(t)}{\mathcal{CE}(t)} \tag{31}$$

Remark 9. *The previous approach can be extended to the carbon intensity measure $\mathcal{CI}(t)$. Moreover, we can use a logarithmic model instead of a linear model:*

$$\ln \mathcal{CE}(t) = \beta_0 + \beta_1 \cdot t + u(t) \tag{32}$$

In this case, we have:

$$\widehat{\mathcal{CE}}(t) = \mathcal{CE}(t_0)\, e^{\hat{\beta}_1 \cdot (t - t_0)} \tag{33}$$

Sequential decarbonization versus self-decarbonization For net zero investment portfolios, we remind that the decarbonization pathway is done with respect to a benchmark at a given reference year t_0. Let $\mathcal{CI}(t, x; \mathcal{F}_s)$ be the carbon intensity of Portfolio x calculated at time t with the information $\mathcal{F}_s$ available at time s. At time t, Portfolio $x(t)$ must satisfy:

$$\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) \leq \left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, t)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right) \tag{34}$$

where $b(t_0)$ is the benchmark at time t_0. We assume that the portfolio is rebalanced at time $t+1$. In this case, we will choose a new portfolio $x(t+1)$ such that:

$$\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) \leq \left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, t+1)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right) \tag{35}$$

We don't have to rebalance the portfolio at time $t+1$ if and only if:

$$\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) \leq \left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, t+1)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right) \tag{36}$$

Therefore, the variation $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)$ between two rebalancing dates can be breakdown into two components:

[23] Generally, h is equal to 1, 2 or 3 years.

1. a self-decarbonization $\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)$ and;
2. an additional decarbonization $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right)$.

The self-decarbonization ratio is then defined as:

$$\begin{aligned} \mathcal{SR}(t+1) &= \frac{\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)}{\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)} \\ &= \frac{\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) - \mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right)}{\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) - \mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right)} \end{aligned} \tag{37}$$

By construction, we have:

$$\mathcal{SR}(t+1) \leq 1 \tag{38}$$

The upper bound is reached when we do not have to rebalance the portfolio. If the carbon intensity of the current portfolio has not changed between t and $t+1$, the self-decarbonization ratio is equal to zero. The worst case is obtained when the carbon intensity has increased, implying that $\mathcal{SR}(t+1) < 0$.

Table 8: Backtesting of net-zero investment portfolios

s	$\mathcal{CI}_\star^s$	Case #1			Case #2		
		$\mathcal{CI}_x^s$	$\mathcal{CI}_x^{s+1}$	$\mathcal{SR}^s$	$\mathcal{CI}_x^s$	$\mathcal{CI}_x^{s+1}$	$\mathcal{SR}^s$
t	100.0	100.0	99.0		100.0	92.0	
$t+1$	93.0	93.0	91.2	14.3%	92.0	85.0	100.0%
$t+2$	86.5	86.5	91.3	27.7%	85.0	80.2	100.0%
$t+3$	80.4	80.4	78.1	−78.7%	80.2	75.0	100.0%
$t+4$	74.8	74.8	74.2	41.1%	74.8	70.0	96.3%
$t+5$	69.6	69.6	70.7	11.5%	69.6	68.9	92.3%
$t+6$	64.7	64.7	62.0	−22.4%	64.7	60.0	14.3%
$t+7$	60.2	60.2	60.0	60.0%	60.0	55.1	100.0%
$t+8$	55.9	55.9	58.3	4.7%	55.1	52.0	100.0%
$t+9$	52.0	52.0	53.5	−61.5%	52.0	47.5	100.0%
$t+10$	48.4	48.4	50.5	−41.7%	47.5	45.5	100.0%

We use the following notations for the labels: $\mathcal{CI}_\star^s$ is equal to $\left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, s)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right)$, $\mathcal{CI}_x^s = \mathcal{CI}\left(s, x(s); \mathcal{F}_s\right)$ is the carbon intensity of Portfolio $x(s)$ at the rebalancing date s, $\mathcal{CI}_x^{s+1} = \mathcal{CI}\left(s+1, x(s); \mathcal{F}_{s+1}\right)$ is the carbon intensity of Portfolio $x(s)$ at the end of the period $[s, s+1]$ before the next rebalancing date $s+1$, and $\mathcal{SR}^s$ is the value of the self-decarbonization ratio for the period $[s-1, s]$.

Let us consider an example to illustrate the concept of self-decarbonization. We assume that the carbon intensity of the benchmark is equal to 200 tCO_2e/\$ mn at the reference date. We begin to reduce the carbon footprint by 50%, targeting a carbon intensity of 100 tCO_2e/\$ mn at time t. Then, we use the following pathway of decarbonization rates: $53.50\%, 56.76\%, \ldots, 73.98\%, 75.80\%$. The targeted carbon intensity is reported in the second column in Table 8. We obtain 93 tCO_2e/\$ mn at time $t+1$, then 86.5, 80.4, etc. until we obtain 48.4 tCO_2e/\$ mn at time $t+10$. We consider a first portfolio. In the third column, we indicate the values taken by $\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)$, $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right)$, etc. The fourth column indicates the carbon intensity of the portfolio at the end of the period: $\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right)$, $\mathcal{CI}\left(t+2, x(t+1); \mathcal{F}_{t+2}\right)$, etc. For instance, we have $\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) = 100$ and $\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) = 99$. The carbon footprint of this portfolio has been reduced during the period $[t, t+1]$, but the self-decarbonization is not enough to reach the target 93 for the rebalancing date $t+1$. Therefore, we need to rebalance the portfolio to impose that $\mathcal{CI}\left(t+1, x\left(t+1\right); \mathcal{F}_{t+1}\right) = 93$. The self-decarbonization ratio is not high and is equal to 14.3%. Sometimes, we can also observe an increase in the carbon footprint during two rebalancing dates. This is the case of portfolio $x\left(t+2\right)$ since its carbon intensity is equal to 86.5 at the beginning of the period and 91.3 at the end of the period. Again, we need to rebalance the portfolio to match the new target, which is 80.4. Case #1 is an example where the net zero pathway is mainly obtained by sequential decarbonization. Case #2 is very interesting because we don't need to rebalance the portfolio most of the time. Indeed, the self-decarbonization is enough for 7 among 10 rebalancing dates.

Remark 10. *In Figures 40 and 41 on page 120, we have created a data visualization about the importance of self-decarbonization (green bars) with respect to sequential decarbonization (blue bars) and negative decarbonization (red bars). This last one occurs when the carbon intensity of the portfolio increases between two rebalancing dates. In Case #1, we see that self-decarbonization is secondary, whereas it dominates in Case #2.*

Remark 11. *The computation of self decarbonization ratios is a first step towards implementing the backtesting of net zero investment portfolios. While backtesting is central to risk management and measurement, it seems that it is completely ignored by net zero processes. However, backtesting analyzes the ex-post validity of a model. Therefore, it is appropriate for validating or not net zero investment processes. Indeed, investors have the right to understand the limits of any net zero portfolio model.*

To maximize the self-decarbonization ratio, we need to model the probability distribution of the estimator $\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right)$. We now understand why carbon trend, temperature rating or carbon momentum have great importance in a net zero process. For instance, the current carbon footprint gives no information about its dynamics. Indeed, if we assume that $\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right) = \mathcal{CI}\left(t, x\left(t\right); \mathcal{F}_t\right)$, we have $\mathbb{E}\left[\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right)\right] > \mathcal{CI}\left(t+1, x\left(t+1\right); \mathcal{F}_{t+1}\right)$ whereas we prefer to have the inequality $\mathbb{E}\left[\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right)\right] \leq \mathcal{CI}\left(t+1, x\left(t+1\right); \mathcal{F}_{t+1}\right)$. Therefore, the real challenge lies in having an idea about the dynamics of the carbon footprint. Even if carbon trend or momentum seems to be simplistic at first sight from a statistical point of view, they are nevertheless relatively objective, they do not depend on sophisticated models and they are easy to understand.

Table 9: Statistics (in %) of carbon momentum $\mathcal{CM}^{\mathcal{L}ong}\left(t\right)$

Statistics	Carbon emissions			Carbon intensity		
	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$
Median	1.7	2.6	2.6	−2.3	−1.7	−1.6
Negative	43.3	37.7	34.9	69.5	66.6	72.0
Positive	56.7	62.3	65.1	30.5	33.4	28.0
$< -10\%$	22.7	17.5	13.3	21.1	14.4	6.5
$< -5\%$	30.0	24.4	19.9	31.5	22.1	13.3
$> +5\%$	34.5	37.6	35.6	11.6	13.2	7.5
$> +10\%$	17.1	17.6	15.0	5.8	6.5	3.3

The table above gives some statistics about carbon momentum. Since we impose to have a track record of 5 years at least, we can compute the long-term carbon momentum for only 69% of issuers that are in the Trucost database. The median value of $\mathcal{CM}^{\mathcal{L}ong}\left(t\right)$ is equal to 1.7% for scope 1, 2.6% when we include scope 2, and 2.6% when we consider upstream scope 3. The median value increases when we incorporate indirect carbon emissions, for both carbon emission and carbon intensity. We cannot compute the carbon trend for scope $1 + 2 + 3$ because the data history for downstream emissions would be too short. The carbon momentum is negative for 34.9% of issuers when we consider $\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$. This means that a majority of issuers have a positive carbon trend. For instance, 15% of issuers have a carbon momentum greater than 10%! If we consider carbon intensity instead of carbon emission, we obtain different results. Indeed, issuers with a negative trend dominate issuers with a positive trend. Therefore, it is easier to build a self-decarbonized portfolio when we consider the carbon intensity measure.

Remark 12. *Considering carbon emissions or carbon intensities gives two very different pictures of the carbon momentum. Even if meeting net zero emission in 2050 implies meeting net zero intensity as well, the pathways to meet this objective are very different.*

4.2 Net zero transition metrics

While the previous section presents the metrics associated with the decarbonization dimension, we need to specify the greenness measures for implementing the transition dimension. However, contrary to the carbon footprint, which is a well-defined concept, greenness is more difficult to assess. In fact, it is a multi-faceted concept. For instance, if one issuer changes its business model so that its new products are carbon efficient, we can measure the issuer's greenness based on the avoided emissions generated by the change of the business model. For other issuers, the greenness can be evaluated by estimating the R&D amount dedicated to green projects. Therefore, we observe a big difference between carbon and transition metrics. Indeed, while it makes sense to compute the carbon footprint of all issuers, the greenness may be indefinite for some issuers, because they have no vocation to participate in the transition. They are neutral and are not exposed to the green business. All these remarks argue in favor of considering simple and homogeneous measures of greenness. For that, we first need to specify a green taxonomy.

4.2.1 Green taxonomy

Definition The purpose of a green financial taxonomy is to define what is green, and its objective is to inform investors about the greenness of their investments. Therefore, they can evaluate whether these levels satisfy or not their expectations. A green taxonomy is all the more important as we observe a strong development of green sentiment among investors (Brière and Ramelli, 2021). Moreover, MiFID II imposes new obligations to take into account sustainable preferences. In this context, the client must determine a minimum proportion that should be invested in environmentally sustainable assets. Therefore, a green taxonomy is necessary for both asset owners and managers.

Buhr and Cormack (2020) explained that "*a taxonomy is a way of organizing knowledge*" usually in a hierarchical order. This top-down approach has many advantages and is well known by investors. For instance, sector classification systems such as GICS or ICB use this method. In a similar way, Alessi and Battiston (2022) considered the NACE classification and estimated a taxonomy alignment coefficient (TAC) for each sector of activity. In this case, we can calculate the green intensity of the portfolio by using the breakdown of the allocation with respect to the NACE classification:

$$\boldsymbol{\mathcal{GI}}(w) = \sum_{j=1}^{m} w_j \cdot \boldsymbol{\mathcal{GI}}_j$$

where w_j is the weight of the j^{th} sector and $\boldsymbol{\mathcal{GI}}_j$ is its green intensity[24]. Nevertheless, we also know that sectoral categories are heterogeneous even when we consider industry or sub-industry

[24] The green intensity is equal to the TAC factor.

levels. In the bottom-up approach, we directly estimate the green intensity at the issuer level and we have:

$$\mathcal{GI}(x) = \sum_{i=1}^{n} x_i \cdot \mathcal{GI}_i$$

where x_i is the weight of the i^{th} issuer and $\mathcal{GI}_i$ is its green intensity. Since the bottom-up approach is more informative than the top-down approach because it operates at the most granular level, it is also more complex as it requires a lot of data. Moreover, we have to estimate these data when they are missing or not mandatory to report.

Remark 13. *From a theoretical point of view, the two approaches are equivalent if we assume that the issuer belongs to a single sector. Indeed, we have $w_j = \sum_{i \in j} x_i$ and:*

$$\mathcal{GI}_j = \frac{\sum_{i \in j} x_i \cdot \mathcal{GI}_i}{\sum_{i \in j} x_i}$$

We deduce that:

$$\begin{aligned}
\mathcal{GI}(w) &= \sum_{j=1}^{m} \left(\sum_{i \in j} x_i \right) \cdot \left(\frac{\sum_{i \in j} x_i \cdot \mathcal{GI}_i}{\sum_{i \in j} x_i} \right) \\
&= \sum_{j=1}^{m} \sum_{i \in j} x_i \cdot \mathcal{GI}_i \\
&= \sum_{i=1}^{n} x_i \cdot \mathcal{GI}_i = \mathcal{GI}(x)
\end{aligned}$$

In a multi-sector framework, the equality $\mathcal{GI}(w) = \mathcal{GI}(x)$ does not hold because $\sum_{j=1}^{m} x_{i,j} \cdot \mathcal{GI}_{i,j} \neq x_i \cdot \mathcal{GI}_i$ where $x_{i,j}$ and $\mathcal{GI}_{i,j}$ are the allocation amount and the green intensity of issuer i in activity j. Another difference between the bottom-up and top-down approaches comes from the fact that the green intensities are calculated with all the issuers of the investment universe in the top-down approach. This is not the case with the bottom-up approach, which only considers the issuers that belong to the portfolio.

As noticed by Buhr and Cormack (2020), a green taxonomy may be restrictive since it tells us nothing about the brownness of the issuer. For example, if an issuer has a green intensity of 30%, this implies that 70% is not green. It may correspond to an issuer whose brown intensity lays between 0% and 70%. Therefore, it is not possible to deduce a brown taxonomy from the green taxonomy. We can only deduce an upper bound:

$$0 \leq \mathcal{BI}_i \leq 1 - \mathcal{GI}_i$$

The advantage of having both a green taxonomy and a brown taxonomy is that we can determine the non-green-brown (or white) intensity $\mathcal{NI}_i$ of the issuer because of the following relationship:

$$\mathcal{BI}_i + \mathcal{NI}_i + \mathcal{GI}_i = 1$$

To avoid a black and white picture of greenness, another solution is to define a green taxonomy, whose range is between 0 and 200% and not between 0 and 100%. For instance, we can propose the following score:

$$\begin{aligned}
\mathcal{GI}_i &= 2 \times \varpi_i^{\mathcal{G}\text{reen}} + 1 \times \left(1 - \varpi_i^{\mathcal{G}\text{reen}} - \varpi_i^{\mathcal{B}\text{rown}}\right) + 0 \times \varpi_i^{\mathcal{B}\text{rown}} \\
&= 1 + \varpi_i^{\mathcal{G}\text{reen}} - \varpi_i^{\mathcal{B}\text{rown}}
\end{aligned}$$

where $\varpi_i^{\mathcal{G}\text{reen}}$ and $\varpi_i^{\mathcal{B}\text{rown}}$ are the proportion of green and brown activities. In this case, if the issuer has 50% in green activities and the remainder in white activities, its green intensity is equal to 150%, whereas the score is equal to 100% if the remainder concerns brown activities.

We have represented the different approaches of an environmental taxonomy in Figure 12. Each type differs in the objective it pursues. For example, the goal of a green-based taxonomy is to identify more strictly green activities to promote them. Therefore, with a green-based taxonomy, investors have no incentive to disinvest from brown activities. This is not the case with a brown-based taxonomy, whose objective is clearly to promote exclusion strategies. On the contrary, a mixed taxonomy recognizes many shades of green and not only one (Carney, 2019). These 3 taxonomy types are the counterpart of ESG investing strategies, that make the difference between selection, exclusion and integration.

Figure 12: Three types of environmental taxonomy

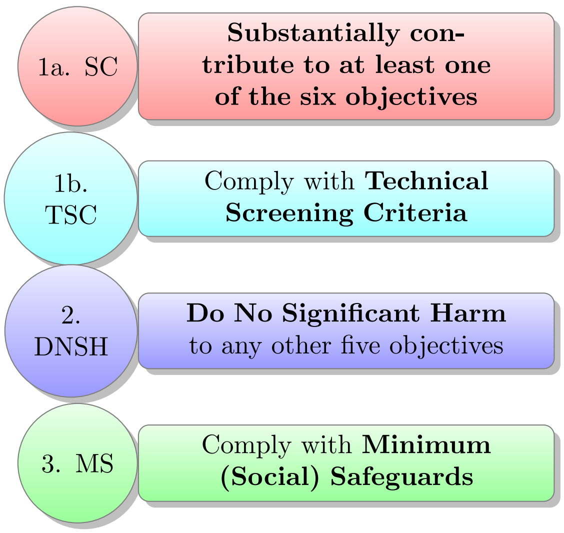
Examples of green/brown taxonomy The most famous example is the European green taxonomy. According to the European Commission[25], the EU taxonomy for sustainable activities is "*a classification system, establishing a list of environmentally sustainable economic activities. [...] The EU taxonomy would provide companies, investors and policymakers with appropriate definitions for which economic activities can be considered environmentally sustainable. In this way, it should create security for investors, protect private investors from greenwashing, help companies to become more climate-friendly, mitigate market fragmentation and help shift investments where they are most needed.*". Developed by the Technical Expert Group (TEG, 2020), the EU green taxonomy defines economic activities which make a substantive contribution to at least one of the following six environmental objectives: (1) Climate change mitigation, (2) Climate change adaptation, (3) Sustainable use and protection of water and marine resources, (4) Transition to a circular economy, (5) Pollution prevention and control, and (6) Protection and restoration of biodiversity and ecosystem. To qualify as sustainable, a business activity must also meet two other criteria. Indeed, the activity must do no significant harm to the other environmental objectives (DNSH constraint) and comply with minimum social safeguards[26] (MS constraint). Figure 12 summarizes the different steps.

The EU taxonomy is not finalized and only concerns the first two objectives as of today (July 2022). Another drawback is that we must use reported data from the companies, implying that estimated data are prohibitive. The use of the EU taxonomy is then limited to assessing the transition dimension in the short term as long as the Corporate Sustainability Reporting Directive (CSRD) is not implemented. In the meantime, we can use proprietary taxonomies developed by data providers. For instance, MSCI has defined its taxonomy for identifying green activities. They are grouped into 6 categories: (1) Alternative energy, (2) Energy efficiency, (3) Green building, (4) Pollution prevention and control, (5) Sustainable agriculture and (6) Sustainable water. The

[25] See the EU website: `https://ec.europa.eu/info/business-economy-euro/banking-and-finance/sustainable-finance_en`.

[26] For example, the UN guiding principles on business and human rights.

Figure 13: EU taxonomy for sustainable activities



green taxonomy of MSCI could be viewed as the first step of the green taxonomy of the European Union without including the DNSH and MS steps.

Remark 14. *In some sense, a brown taxonomy is included in the EU taxonomy since the TSC and DNSH criteria are related to brown activities. If we consider data providers, brown activities are generally limited to the following sectors: coal, oil/petroleum, and gas.*

4.2.2 Static measures of greenness

There are several ways to compute the green intensity. This is why we observe some significant differences between data providers. One method is to translate the 3-step approach of the EU taxonomy into the following equation:

$$\boldsymbol{\mathcal{GI}} = \frac{\boldsymbol{\mathcal{GR}}}{\boldsymbol{\mathcal{TR}}} \cdot (1 - \mathcal{P}) \cdot \mathbb{1}\left\{\mathcal{S} \geq \mathcal{S}^{-}\right\} \tag{39}$$

where $\boldsymbol{\mathcal{GR}}$ is the green revenues deduced from the objectives, $\boldsymbol{\mathcal{TR}}$ is the total revenues, $\mathcal{P}$ is the penalty coefficient reflecting the DNSH constraint, $\mathcal{S}$ is the minimum safeguard score and $\mathcal{S}^{-}$ is the threshold. The first term is a proxy of the turnover KPI and corresponds to the green revenue share:

$$\boldsymbol{\mathcal{GRS}} = \frac{\boldsymbol{\mathcal{GR}}}{\boldsymbol{\mathcal{TR}}} \tag{40}$$

By construction, we have $0 \leq \boldsymbol{\mathcal{GRS}} \leq 1$. This measure is then impacted by the DNSH coefficient. If the penalty coefficient is equal to zero, the green activities of the issuer do not significantly harm the other objectives and we have $\boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}}$. Otherwise, the green intensity satisfies $0 \leq \boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}} \cdot (1 - \mathcal{P}) \leq \boldsymbol{\mathcal{GRS}}$. Finally, the indicator function $\mathbb{1}\left\{\mathcal{S} \geq \mathcal{S}^{-}\right\}$ is a binary all-or-nothing variable. It is equal to one if the firm complies with minimum social safeguards. Otherwise, the green intensity is equal to zero if the firm doesn't pass this materiality test. It follows that an upper bound of the green intensity is the green revenue share since we have $\boldsymbol{\mathcal{GI}} \leq \boldsymbol{\mathcal{GRS}}$. In what follows, we assume that $\boldsymbol{\mathcal{GI}} \approx \boldsymbol{\mathcal{GRS}}$, implying that our results overestimate the green taxonomy of investments. Moreover, it is easier to find gross green revenue shares than net revenue shares aligned with the EU taxonomy.

Table 10: Statistics in % of green revenue share (MSCI ACWI IMI)

Category	Frequency $\mathbf{F}(x)$				Quantile $\mathbf{Q}(\alpha)$				Mean	
	0	25%	50%	75%	75%	90%	95%	Max	Avg	Wgt
(1)	9.82	1.47	0.96	0.75	0.00	0.00	2.85	100.00	1.36	0.77
(2)	14.10	1.45	0.65	0.31	0.00	1.25	6.12	100.00	1.39	3.50
(3)	4.84	1.68	1.02	0.31	0.00	0.00	0.00	100.00	1.16	0.51
(4)	4.79	0.30	0.10	0.06	0.00	0.00	0.00	99.69	0.32	0.22
(5)	1.00	0.39	0.20	0.09	0.00	0.00	0.00	98.47	0.26	0.10
(6)	4.75	0.28	0.11	0.05	0.00	0.00	0.00	99.98	0.29	0.14
Total	27.85	5.82	3.17	1.68	0.42	11.82	30.36	100.00	4.78	5.24

In Table 10, we report some descriptive statistics about the green revenue share based on the MSCI database. We use the MSCI ACWI IMI universe with 9283 issuers. For each category[27], we compute the frequency $\mathbf{F}(x) = \Pr\{\boldsymbol{\mathcal{GRS}} > x\}$, the statistical quantile $\mathbf{Q}(\alpha) = \inf\left\{x : \Pr\{\boldsymbol{\mathcal{GRS}} \leq x\} \geq \alpha\right\}$, the average $\overline{\boldsymbol{\mathcal{GRS}}} = n^{-1}\sum_{i=1}^{n}\boldsymbol{\mathcal{GRS}}_i$ and the weighted mean $\boldsymbol{\mathcal{GRS}}(b) = \sum_{i=1}^{n} b_i \cdot \boldsymbol{\mathcal{GRS}}_i$ where b_i is the weight of Issuer i in the MSCI ACWI IMI benchmark. For instance, 9.82% of issuers have a green revenue share that concerns alternative energy. This figure becomes less than 1% if we consider a green revenue share greater than 50%. The average value is equal to 1.36% whereas the weighted value is equal to 0.77%. This indicates a small cap bias. For energy efficiency, the average is lower than the weighted mean, implying a bias towards big companies. If we consider the total green revenue share, 27.85% have a positive figure and only 3.17% have a figure greater than 50%. The 90% quintile is equal to 11.82%. Therefore, we notice a high positive skewness for the distribution. The green revenue share is then located in a small number of companies.

4.2.3 Dynamic measures of greenness

A first approach to define a dynamic measure of greenness is to estimate the trend of the green intensity (or the green revenue share).

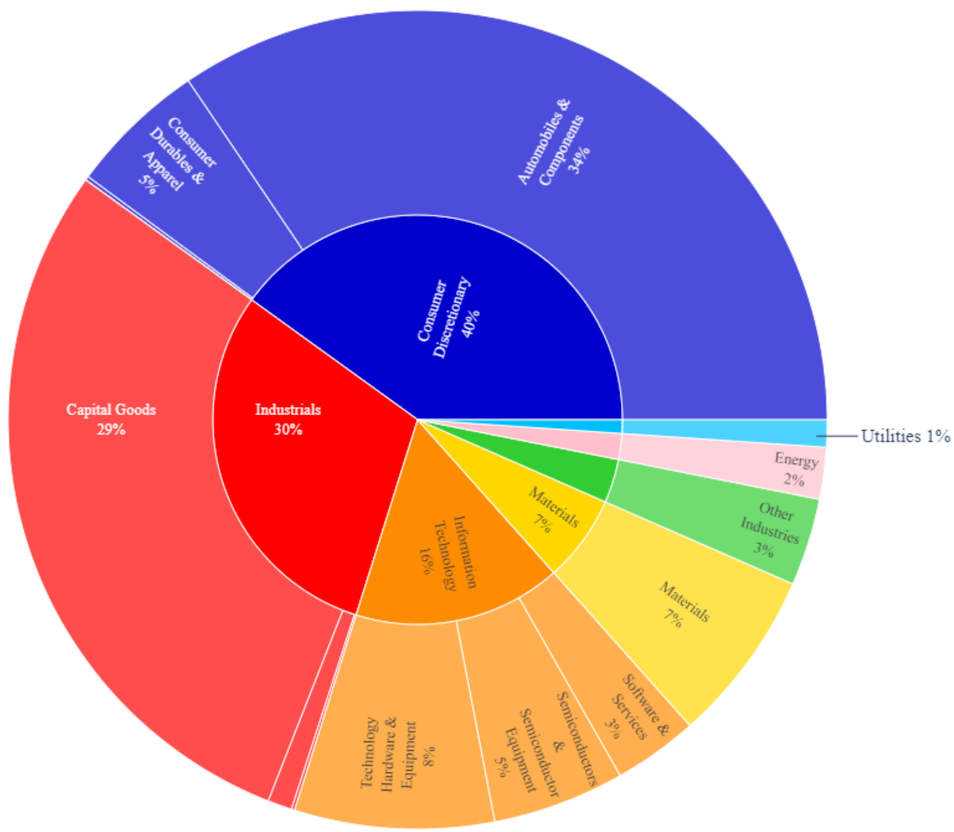
$$\boldsymbol{\mathcal{GI}}(t) = \gamma_0 + \gamma_1 \cdot t + v(t)$$

We can then build the same dynamic measures as those defined for the carbon metrics: green trend, green velocity and green momentum. The current issue is that we do not have a long historical time series of green revenue shares. Instead of estimating $\widehat{\boldsymbol{\mathcal{GI}}}(t) = \boldsymbol{\mathcal{GI}}(t_0) + \hat{\gamma}_1 \cdot (t - t_0)$, we can use a proxy or a KPI that contains information about the future green intensity of the issuer. A first indicator may be the green capex. The rationale is the following. According to IEA (2021), "*almost half of the emissions savings needed in 2050 to reach net zero emissions rely on technologies that are not yet commercially available*". All the climate scenarios describe the same need, that is a significant level of green investment in clean transportation, clean energy, energy storage, or carbon capture and storage to name a few. Therefore, it does make sense to assess the current green investment, which can be measured by green capex. Unfortunately, very few companies are disclosing it at this time. For example, the green capex metric provided by Reuters Eikon covers barely 100 companies in the MSCI World. There is increasing pressure on companies to disclose their green capex, and the data availability will soon be improved[28].

[27]We remind them: (1) Alternative energy, (2) Energy efficiency, (3) Green building, (4) Pollution prevention and control, (5) Sustainable agriculture and (6) Sustainable water.

[28]For example, the disclosure of aligned capex is required for European companies under the EU Green Taxonomy.

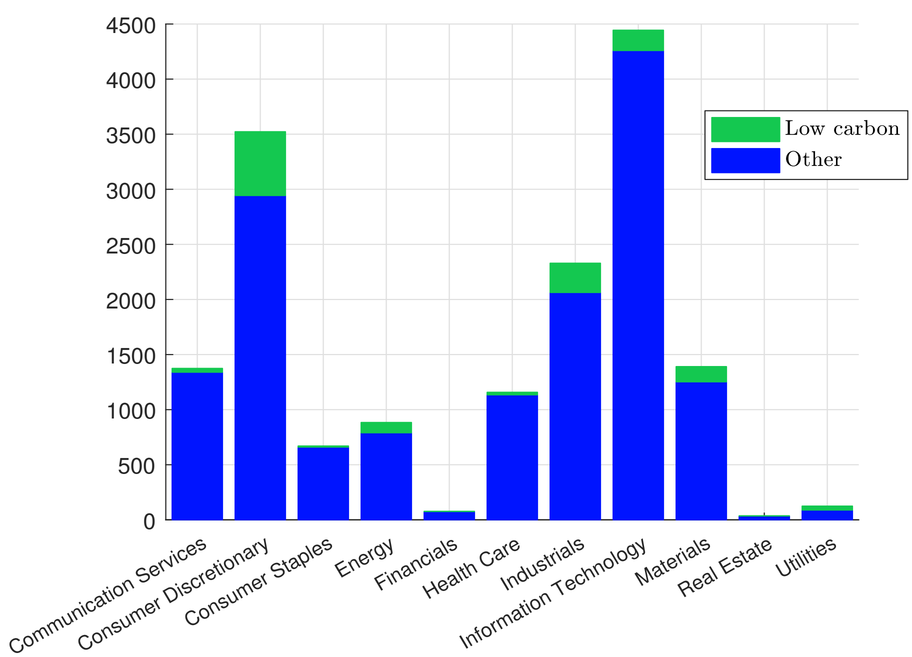
Figure 14: Patent breakdown per GICS sector and industry



Low-carbon patents are another measure of a company's research effort on climate solutions. The European Patent Office (EPO) has developed a classification scheme for climate mitigation and adaptation technologies, which allows for low-carbon patents identification. Green capex and low-carbon patents meet the same need since they provide a forward-looking measure of green revenues. For example, it took between 10 and 30 years between the prototype and the mass market for LEDs or lithium-ion batteries development (European Patent Office, 2021), leading to a large lag between forward-looking measures and green revenues. However, green capex and low-carbon patents have many dissimilarities. Green capex is a leading indicator of a company's ability to innovate, as the patent filing process takes between one and three years. On the one hand, green capex spending does not indicate whether these funds have resulted in patent registration or commercialization. On the other hand, a company may decide not to file a patent, and benefit from its innovation. Moreover, a company can hold a patent and not exploit it. These two metrics are therefore complementary. The advantage of low-carbon patents metric over green capex is data availability. It covers 80% of the companies of the MSCI World, representing 93% of the market cap. Low-carbon patents are mostly filed by companies belonging to polluting sectors (e.g., Automobile and Capital Goods), with the exception of Information Technology. If the Utilities sector represents a small share of the low-carbon patents, almost 24% of its patents are green.

Remark 15. *Figures 14 and 15 illustrate this phenomenon. The Automotive industry files more than 85% of the low-carbon patents in the Consumer Discretionary sector, with the latter holding 40% of all low-carbon patents. The Automotive industry thereby accounts for more than one-third of the total number of low-carbon patents held by companies of the MSCI World index. However, although this sector leads by a wide margin in terms of the number of low-carbon patents, a study of the share that this represents in all of its patents paints a different picture. Indeed, despite filing the largest number of low-carbon patents, they represent only 6.3% of the sector's patents while the Utilities sector is in the exact opposite situation. Thus, one might consider these two indicators when constructing a portfolio to get a more balanced picture of companies low-carbon innovation.*

Figure 15: Average number of patents per GICS sector



5 Portfolio optimization : a general framework

Implementations of net zero portfolios generally build on including climate risk metrics in modern portfolio optimization. This approach implies the use of a business-as-usual benchmark and does not fundamentally change the traditional framework of portfolio construction as introduced by Harry Markowitz[29]. His findings are at the core of portfolio management and the mean-variance optimization (MVO) theory for portfolio selection is certainly one of the most famous methods used in asset management. MVO is closely related to the Quadratic Programming (QP) problem (Markowitz, 1956), hence deriving the solution of the latter permits solving the Markowitz assets allocation model. This portfolio allocation model only needs two input parameters : expected returns and the covariance matrix of assets returns. After estimating the input parameters, the optimization is done as if these quantities were perfectly certain, implying that estimation errors are introduced into the allocation process[30]. For some specific asset classes, such as fixed-income for example, the estimation of returns and particularly of risk, is however far more complex than for equities. Recent development of the 2008 and the European debt crisis has strongly challenged traditional bond management and highlighted the need to take into account various factors such as credit and interest-rate risk. Such distinctions, alongside the relatively bigger size of bond indexes forced investors to manage risk in a different manner between these asset classes.

5.1 Mean-Variance optimization

Seventy years ago, Markowitz has formalized the problem of portfolio optimization (Markowitz, 1952). For him, "*the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing*". Indeed, Markowitz showed that an efficient portfolio is the portfolio that maximizes the expected return for a given level of risk (corresponding to the variance of portfolio return) or a portfolio that minimizes the risk for a given level of expected return. Even if this framework has been extended to many other allocation problems (index sampling, turnover management, etc.), the mean-variance model remains the optimization approach that is the most widely used in quantitative finance.

Assets Allocation problem We consider a universe of n assets. Let $x = (x_1, \ldots, x_n)$ be the vector of weights in the portfolio. We assume that the portfolio is fully invested meaning that $\sum_{i=1}^{n} x_i = \mathbf{1}_n^\top x = 1$. We denote $R = (R_1, \ldots, R_n)$ as the vector of asset returns where R_i is the return of asset i. The return of the portfolio is then equal to $R(x) = \sum_{i=1}^{n} x_i R_i = x^\top R$. Let $\mu = \mathbb{E}[R]$ and $\Sigma = \mathbb{E}\left[(R-\mu)(R-\mu)^\top\right]$ be the vector of expected returns and the covariance matrix of asset returns. The expected return of the portfolio is equal to:

$$\mu(x) = \mathbb{E}\left[R(x)\right] = x^\top \mu$$

whereas its variance is equal to:

$$\sigma^2(x) = \mathbb{E}\left[\left(R(x) - \mu(x)\right)\left(R(x) - \mu(x)\right)^\top\right] = x^\top \Sigma x$$

Markowitz (1952) formulated the investor's financial problem as follows:

1. Maximizing the expected return of the portfolio under a volatility constraint (σ-problem):

$$\max \mu(x) \quad \text{u.c.} \quad \sigma(x) \leq \sigma^\star \tag{41}$$

[29]This section largely uses the formulation presented in Roncalli (2013). For the reader to get a broader view of portfolio optimization, we highly recommend to refer to the book.

[30]Literature abounds of regularization methods for stabilizing the optimization, from the simplest to the most complicated (Roncalli, 2013).

2. Or minimizing the volatility of the portfolio under a return constraint (μ-problem):

$$\min \sigma(x) \quad \text{u.c.} \quad \mu(x) \geq \mu^{\star} \tag{42}$$

Markowitz's bright idea was to consider a quadratic utility function:

$$\mathcal{U}(x) = x^{\top}\mu - \frac{\phi}{2}x^{\top}\Sigma x$$

where $\phi \geq 0$ is the risk aversion. Since maximizing $\mathcal{U}(x)$ is equivalent to minimizing $-\mathcal{U}(x)$, the Markowitz problems (41) and (42) can be cast into a QP problem[31]:

$$\begin{aligned} x^{\star}(\gamma) &= \arg\min_{x} \frac{1}{2}x^{\top}\Sigma x - \gamma x^{\top}\mu \\ &\text{s.t.} \quad \mathbf{1}_n^{\top}x = 1 \end{aligned} \tag{43}$$

where $\gamma = \phi^{-1}$. Therefore, solving the μ-problem or the σ-problem is equivalent to finding the optimal value of γ such that $\mu(x^{\star}(\gamma)) = \mu^{\star}$ or $\sigma(x^{\star}(\gamma)) = \sigma^{\star}$. Moreover, it is easy to include bounds on the weights, inequalities between asset classes and climate-related constraints.

5.2 Quadratic programming

5.2.1 Primal formulation

A quadratic programming (QP) problem is an optimization problem with a quadratic objective function and linear inequality constraints:

$$\begin{aligned} x^{\star} &= \arg\min_{x} \frac{1}{2}x^{\top}Qx - x^{\top}R \\ &\text{s.t.} \quad Sx \leq T \end{aligned} \tag{44}$$

where x is a $n \times 1$ vector, Q is a $n \times n$ matrix and R is a $n \times 1$ vector. We note that the system of constraints $Sx \leq T$ allows us to specify linear equality constraints[32] $Ax = B$ or box constraints $x^{-} \leq x \leq x^{+}$. Most numerical packages then consider the following formulation:

$$\begin{aligned} x^{\star} &= \arg\min_{x} \frac{1}{2}x^{\top}Qx - x^{\top}R \\ &\text{s.t.} \quad \left\{ \begin{array}{l} Ax = B \\ Cx \leq D \\ x^{-} \leq x \leq x^{+} \end{array} \right. \end{aligned} \tag{45}$$

because the problem (45) is equivalent to the canonical problem (44) with the following system of linear inequalities:

$$\left[\begin{array}{c} -A \\ A \\ C \\ -I_n \\ I_n \end{array} \right] x \leq \left[\begin{array}{c} -B \\ B \\ D \\ -x^{-} \\ x^{+} \end{array} \right]$$

If the space Ω defined by $Sx \leq T$ is non-empty and if Q is a symmetric positive definite matrix, the solution exists because the function $f(x) = \frac{1}{2}x^{\top}Qx - x^{\top}R$ is convex. In the general case where Q is a square matrix, the solution may not exist.

Remark 16. *It is then obvious that a large part of the success of the Markowitz framework lies on the QP trick. Indeed, Problem (43) corresponds to the QP problem (45) where $Q = \Sigma$, $R = \gamma\mu$, $A = \mathbf{1}_n^{\top}$ and $B = 1$.*

[31] This transformation is called the QP trick.

[32] This is equivalent to impose that $Ax \geq B$ and $Ax \leq B$.

5.2.2 Dual formulation

The Lagrange function is equal to:

$$\mathcal{L}(x;\lambda)=\frac{1}{2}x^\top Qx-x^\top R+\lambda^\top(Sx-T)$$

where λ is the vector of Lagrange coefficients associated with the constraint $Sx\leq T$.We deduce that the dual problem is defined by:

$$\begin{array}{rcl}\lambda^\star & = & \arg\max_\lambda\left\{\inf_x\mathcal{L}(x;\lambda)\right\}\\ & \text{s.t.} & \lambda\geq 0\end{array}$$

We note that $\partial_x\,\mathcal{L}(x;\lambda)=Qx-R+S^\top\lambda$. The solution to the equation $\partial_x\,\mathcal{L}(x;\lambda)=0$ is then $x=Q^{-1}\left(R-S^\top\lambda\right)$. We finally obtain:

$$\begin{aligned}\inf_x\mathcal{L}(x;\lambda) &= \frac{1}{2}\left(R^\top-\lambda^\top S\right)Q^{-1}\left(R-S^\top\lambda\right)-\left(R^\top-\lambda^\top S\right)Q^{-1}R+\\ &\quad\ \lambda^\top\left(SQ^{-1}\left(R-S^\top\lambda\right)-T\right)\\ &= \frac{1}{2}R^\top Q^{-1}R-\lambda^\top SQ^{-1}R+\frac{1}{2}\lambda^\top SQ^{-1}S^\top\lambda-R^\top Q^{-1}R+\\ &\quad\ 2\lambda^\top SQ^{-1}R-\lambda^\top SQ^{-1}S^\top\lambda-\lambda^\top T\\ &= -\frac{1}{2}\lambda^\top SQ^{-1}S^\top\lambda+\lambda^\top\left(SQ^{-1}R-T\right)-\frac{1}{2}R^\top Q^{-1}R\end{aligned}$$

We deduce that the dual program is another quadratic programming problem:

$$\begin{array}{rcl}\lambda^\star & = & \arg\min_\lambda\frac{1}{2}\lambda^\top\bar{Q}\lambda-\lambda^\top\bar{R}\\ & \text{s.t.} & \lambda\geq 0\end{array}\tag{46}$$

where $\bar{Q}=SQ^{-1}S^\top$ and $\bar{R}=SQ^{-1}R-T$.

Remark 17. *This duality property is very important for some optimization methods that extensively use it for defining the solution in simple cases.*

5.2.3 Numerical algorithms

There is a substantial literature on the methods for solving QP problems (Gould and Toint, 2000). The research begins in the 1950s with different key contributions: Frand and Wolfe (1956), Markowitz (1956), Beale (1959) and Wolfe (1959). Nowadays, QP problems are generally solved using three approaches: active set methods, gradient projection methods and interior point methods. All these algorithms are implemented in standard mathematical programming languages (Matlab, Matematica, Python, Gauss, R, etc.). This explains the success of QP problems since 2000s, because they can be easily and rapidly solved.

5.3 Optimization in presence of a benchmark

The previous framework can be extended to other portfolio allocation problems. However, from a numerical point of view, the underlying idea is to always find an equivalent QP formulation.

We now consider a benchmark b that can be the current portfolio (active management) or an index portfolio (passive management). A typical example of weighting scheme for an equity benchmark portfolio is including each company i in amounts that corresponds to their market capitalization. This so-called *market cap* or *cap weighted* portfolio has for weights $b_i = \frac{\mathcal{MC}_i}{\sum_{i=1}^{n}\mathcal{MC}_i}$, where $\mathcal{MC}_i$ denotes the market capitalization of company i. We note $\mu\left(x \mid b\right) = \left(x - b\right)^\top \mu$ as the expected excess return and $\sigma\left(x \mid b\right) = \sqrt{\left(x - b\right)^\top \Sigma \left(x - b\right)}$ as the tracking error volatility of portfolio x with respect to benchmark b. The objective function corresponds to a trade-off between minimizing the tracking error volatility and maximizing the expected excess return (or the alpha):

$$f\left(x \mid b\right) = \frac{1}{2}\sigma^2\left(x \mid b\right) - \gamma\mu\left(x \mid b\right) \tag{47}$$

We can show that the equivalent QP problem is[33]:

$$x^\star\left(\gamma\right) = \arg\min_x \frac{1}{2}x^\top \Sigma x - \gamma x^\top \tilde{\mu}$$

where $\tilde{\mu} = \mu + \gamma^{-1}\Sigma b$ is the regularized vector of expected returns. Therefore, portfolio allocation with a benchmark can be viewed as a regularization of the MVO problem and is solved using a QP numerical algorithm. In what follows, we will in particular focus of finding the optimal portfolio that minimizes the tracking error volatility with respect to some climate constraints. This approach is equivalent to imposing $\gamma = 0$ or $\mu = \mathbf{0}_n$ in (47) and is extensively used in passive management (Andersson *et al.*, 2016).

Estimation of input parameters Minimizing the tracking error then requires to estimate the covariance matrix Σ. A first approach would be to use the empirical covariance of returns. A second approach is to use a multivariate risk factor model. For the sake of simplicity, we use a single factor model with market returns as the factor. Thus, the single factor model can be written as :

$$R_i\left(t\right) = \alpha_i + \beta_i R_M\left(t\right) + \varepsilon_i\left(t\right) \tag{48}$$

where $R_M\left(t\right)$ are the market returns over time and $\varepsilon_i\left(t\right)$ is the specific risk of asset i. By denoting $\Omega = \operatorname{var}\left(R_M\right)$ and $D = \operatorname{var}\left(\varepsilon_1, \ldots, \varepsilon_n\right) = \operatorname{diag}\left(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_n^2\right)$ the variances over time, we obtain $\Sigma = B\Omega B^\top + D$.

5.4 The case of bonds

The failure of historical returns As for equities, bonds indexing was for a long time based on market-capitalizations for weights construction. Although this weighting scheme was successful for many years, it obviously suffered from not taking into account a fundamental risk factor : the ability to pay the principal and interests on the requested period of time. As underlined by Roncalli (2013), "*an issuer facing financial hardship and trapped in a debt spiral to remain solvent would see its index weight increase until the whole mechanism collapses and an exclusion from the index occurs* ". Thus, index providers preferred debt market price rather than debt notional in order to overcome this issue. This however raises the problem of bonds market data reliability. Indeed, bonds are generally traded in an over-the-counter market, therefore and contrary to the equity market, the notion of closing prices is not well defined for the bond market. For this reason, the volatility of prices return, a measure traditionally used for equity risk, does not quite apply to the bonds. In practice, this makes it also difficult to access information on bonds' mark-to-market price and the information may even differ regarding the provider. In addition, historical prices return suffers when attempting to scale to larger portfolios such as fixed-income portfolios. Quadratic Programming also fails to scale with such large portfolios because of the great dimension of the covariance matrix. In consequence, we will take a different approach for bond portfolio optimization in section 6.1.2, involving fixed-income specific risks metrics and linear programming to overcome the obstacle of dimensionality.

[33] See Appendix A.2.1 on page 98.

5.4.1 Managing risk of bond portfolios

The historical returns approach is all the more not useful in the fixed-income world because there are very obvious market factors affecting all assets, very large number of individual securities, and because of time dependent characteristics of the assets. For example, volatility would not only measure the specific risk of an issuer but rather also reflects movements in the interest rate yield curves. Even considering a single yield curve would not solve this issue as bonds in a portfolio would react accordingly to their differing sensitivities (duration, convexity etc.) and the same conclusion applies when price returns are used to measure bonds' co-movements through a correlation matrix. Hence, for fixed income securities, risk measurement is fundamentally a more complex task than it is for securities such as equities. There are a lot more moving parts. In what follows, we mainly focus on the two components that constitute the risk of the defaultable bond in Bruder *et al.* (2011) and Roncalli (2013) : interest-rate risk and credit risk.

Tracking risk measure Since volatility of returns is not as reliable for fixed-income than for equity, computing the traditional tracking error when constructing a portfolio $x = (x_1, \dots x_n)$ with respect to a benchmark $b = (b_1, \dots b_n)$ is not as informative as we would expect. Moreover, computing a covariance matrix for bond portfolios is a truly difficult exercise because of the large size of bond indexes and the fact that volatility would encompass many factors. Thereby, we replace this metrics with a one that only measures the difference in weights between two portfolios. To assess such a discrepancy between portfolio x and the benchmark b we use the active share :

$$\begin{aligned} \mathcal{AS}\left(x \mid b\right) &= \frac{1}{2}\left\|x-b\right\|_1 \\ &= \frac{1}{2}\sum_{i=1}^{n}\left|x_i-b_i\right| \end{aligned}$$

Modified Duration We assess interest-rate risk through modified duration. Duration, as defined by Macaulay, is a weighted average of the times remaining to various cash flows where the weights are the relative present values of those cash flows. We can write this as :

$$Duration = \sum_{i=1}^{n}\frac{t_i B\left(t,t_i\right) C_{t_i}}{P} \tag{49}$$

where t_i is the time until payment i-th, n the number of remaining payments, $B\left(t,t_i\right)$ is the price of a \$1 zero-coupon that matures t_i periods from now (discount factor), C_{t_i} the bond cash flow in t_i periods from now (the last cash-flow includes the principal) and P the market price of the bond[34]. However, to get a useful measure of a bond's price sensitivity to a change in yield, asset managers prefer modified duration :

$$MD = -\frac{1}{P}\frac{\partial P}{\partial y} \tag{50}$$

where P is the present value of all cash payments from the bonds and y represents the yield. In this form we have the following approximation :

$$-MD\cdot\Delta y \approx \frac{\Delta P}{P}$$

and modified duration therefore expresses the effect of a 1% change in interest rate on a bond price in percent. When constructing a portfolio $x = (x_1, \dots x_n)$, an investor might want to keep it duration neutral with respect to a benchmark $b = (b_1, \dots b_n)$ which rewrites as:

$$\sum_{i=1}^{n}\left(x_i-b_i\right)\cdot MD_i = 0$$

[34] We have $P = \sum_{i=1}^{n} B\left(t,t_i\right) C_{t_i}$

Spread and credit risk To prevent a bond portfolio from high credit risk exposure, asset managers often refer to Duration-Times-Spread (DTS). Following the previous price sensitivity intuition, if we define a spread duration D, the approximation :

$$-D \cdot \Delta s \approx \frac{\Delta P}{P} \tag{51}$$

holds. That is, we can quantify the bond return strictly due to its spread. The idea of Ben Dor *et al.* (2007) is then to rewrite this approximation as :

$$\frac{\Delta P}{P} \approx -D \cdot s \cdot \frac{\Delta s}{s} \tag{52}$$

which is strictly equivalent and where $D \cdot s$ is called *Duration-Times-Spread*. Nonetheless, equations (51) and (52) lead to two distinct representations of excess return volatility due to spread :

$$\sigma_{return} = D \cdot \sigma_{spread} \tag{53}$$

$$\sigma_{return} = \text{DTS} \cdot \sigma_{spread}^{relative} \tag{54}$$

That is, based upon empirical evidence, they show how changes in spreads are not parallel but rather linearly proportional to the level of spread. Therefore the spread volatility of a sector evolves proportionally with its spread level and volatility of returns is proportional to DTS. From these findings, asset managers widely use the DTS metrics as a measure of credit risk and we define the active credit risk of a portfolio x in the presence of a benchmark as :

$$\text{DTS}\left(x \mid b\right) = \sum_{s=1}^{n_{Sector}} \left| \sum_{i \in s} (x_i - b_i) \cdot \underset{i}{\text{DTS}} \right| \tag{55}$$

This measure is decomposed as the sum of discrepancies over every sector of the portfolio to take into account similarities in bonds' characteristics. To assess the global credit quality of the bond portfolio with respect to the benchmark, an asset manager might want to keep it's weights relatively neutral in regards of credit ratings :

$$\forall j, \sum_{i \in \mathcal{R}ating(j)} (x_i - b_i) = 0$$

where index j runs through all the rating categories.

5.4.2 A linear programming problem

Finally, we propose a minimization problem in order to construct a bond portfolio. Consider the following minimization :

$$x^\star(\boldsymbol{\mathcal{R}}) = \arg\min \varphi \, \text{DTS}\left(x \mid b\right) + \mathcal{AS}\left(x \mid b\right) \tag{56}$$

under constraints of modified duration neutral and rating neutral weights in order to manage both interest and credit risk while looking for a low discrepancy between the benchmark and a net zero bond portfolio. φ is a trade-off coefficient between DTS and $\mathcal{AS}$ components. (56) is a particular form of a linear programming problem which is usually written as :

$$\begin{aligned} x^\star &= \arg\min_x c^\top x \\ &\text{s.t.} \left\{ \begin{array}{l} Ax = B \\ Cx \leq D \\ x^- \leq x \leq x^+ \end{array} \right. \end{aligned} \tag{57}$$

The particularity of problem (56) comes from the non-linearity induced by the absolute values in the objective function and the constraints. However as shown by Shanno and Weil (1971) and Vanderbei (2008), it can easily be put in the form of (57) by reformulating the objective function and adding constraints. From a numerical point of view, solutions are generally computed following the simplex algorithm or the method of interior points. Many pythons packages such as SciPy (Virtanen *et al.*, 2008) and CVXPY (Diamond and Boyd, 2016) implement these methods and made linear programming really versatile as some directly handle our objective function form.

6 Net zero investing

Following Le Guenedal and Roncalli (2022), we consider the construction of net zero investment portfolios based on benchmark optimization. The underlying idea is to modify an existing benchmark portfolio by introducing net zero features. This top-down approach, which is based on asset allocation, is used extensively in passive management. However, it is not appropriate in active management, whose bottom-up approach is based on asset selection. While the top-down approach can be easily replicated, the bottom-up approach is difficult to backtest because it depends on too many discretionary choices, including the number of selected assets, the scoring system, the weighting scheme, and the timing of rebalancing. The top-down approach is more standardized and replicable. In what follows, we therefore consider the top-down approach to show how net zero investing differs from portfolio decarbonization. We also consider a core-satellite framework, which is more appropriate for bottom-up approaches and strategic asset allocation.

6.1 Decarbonization approach

In what follows, we distinguish equity portfolios from bond portfolios because the objective function is not the same due to two different definitions of the tracking risk. For equity portfolios, the benchmark is the MSCI World index, whereas we use the Bloomberg Global Investment Grade Corporate Bond index for bond portfolios. The MSCI World index includes large and mid cap representations across developed markets countries. As of June 2022, the index was made of 1 513 constituents. For its part, the Bloomberg Global Investment Grade Corporate Bond Index includes bonds from developed and emerging market issuers that meet various criteria (AAA to BBB ratings, minimum amount outstanding, at least one year to final maturity, etc.). This index is made of around 18 000 securities.

6.1.1 Equity portfolios

Benchmark analysis Let $b = (b_1, \ldots, b_n)$ be the weights of the stocks that belong to the benchmark. Its carbon intensity is given by its weighted average:

$$\mathcal{CI}(b) = \sum_{i=1}^{n} b_i \cdot \mathcal{CI}_i \tag{58}$$

where $\mathcal{CI}_i$ is the carbon intensity of stock i. If we focus on the carbon intensity for a given sector, we use the following formula:

$$\mathcal{CI}\left(\mathcal{S}ector_j\right) = \frac{\sum_{i \in \mathcal{S}ector_j} b_i \cdot \mathcal{CI}_i}{\sum_{i \in \mathcal{S}ector_j} b_i} \tag{59}$$

In Table 11, we report the carbon intensity of the MSCI World index and its sectors. We obtain 130 tCO_2e/\$ mn for scope 1, 163 tCO_2e/\$ mn if we include scope 2, 310 tCO_2e/\$ mn if we add upstream scope 3, and finally 992 tCO_2e/\$ mn if we consider the full scope 3. We notice a large cap bias because the MSCI World equally-weighted portfolio shows higher figures. We also observe a high discrepancy between sectors. Low-carbon sectors are Communication Services, Financials, Health Care and Information Technology, whereas high-carbon sectors are Energy, Materials and Utilities. We foresee that decarbonizing a portfolio implies reducing the exposure to high-carbon sectors and increasing the exposure to low-carbon sectors. For Industrials and Consumer Staples, the sector allocation will depend on the choice of the scope.

Table 11: Carbon intensity in tCO_2e/\$ mn per GICS sector (MSCI World, June 2022)

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	2	28	134	172
Consumer Discretionary	23	65	206	590
Consumer Staples	28	55	401	929
Energy	632	698	1 006	6 823
Financials	13	19	52	244
Health Care	10	22	120	146
Industrials	111	130	298	1 662
Information Technology	7	23	112	239
Materials	478	702	1 113	2 957
Real Estate	22	101	167	571
Utilities	1 744	1 794	2 053	2 840
MSCI World	130	163	310	992
MSCI World EW	168	211	391	1 155

We can compute the risk contribution of each sector as follows:

$$\mathcal{RC}\left(\mathcal{S}ector_j\right) = \frac{\sum_{i \in \mathcal{S}ector_j} b_i \cdot \mathcal{CI}_i}{\mathcal{CI}\left(b\right)} \tag{60}$$

Results are reported in Table 12. For example, Consumer Services represents 7.58% of the nominal allocation, but only 0.14% of the carbon allocation if we consider scope 1. If we focus on the first two scopes, Utilities is the main contributor, followed by Energy and Materials. By including upstream scope 3 emissions, the contribution of Consumer Staples becomes significant. We also notice that the Utilities contribution has strongly been reduced whereas the Industrials contribution increases when we consider the three scopes.

Table 12: Sectoral contribution in % (MSCI World, June 2022)

Sector	Index	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	7.58	0.14	1.31	3.30	1.31
Consumer Discretionary	10.56	1.87	4.17	6.92	6.21
Consumer Staples	7.80	1.68	2.66	10.16	7.38
Energy	4.99	24.49	21.53	16.33	34.37
Financials	13.56	1.33	1.58	2.28	3.34
Health Care	14.15	1.12	1.92	5.54	2.12
Industrials	9.90	8.38	7.83	9.43	16.38
Information Technology	21.08	1.13	3.03	7.57	5.06
Materials	4.28	15.89	18.57	15.48	12.93
Real Estate	2.90	0.48	1.81	1.57	1.65
Utilities	3.21	43.47	35.59	21.41	9.24

It is also important to take into account the carbon momentum metric, as shown in Table 13. We use the aggregation method described in Appendix A.2.2 on page 98. On average, the carbon momentum of the MSCI World index is negative and only 25% of issuers have positive momentum.

Table 13: Carbon (intensity) momentum $\mathcal{CM}^{\mathcal{L}ong}$ in % by sector (MSCI World, June 2022)

Sector	Average $\mathcal{CM}_x^{\mathcal{L}ong}$			Frequency $\mathcal{CM}_i^{\mathcal{L}ong} > 0$		
	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$
Communication Services	−7.3	0.7	0.9	29.5	40.9	44.3
Consumer Discretionary	−0.1	−0.3	−1.1	16.3	23.5	15.7
Consumer Staples	−5.0	−4.4	−2.2	17.8	17.8	15.8
Energy	2.3	2.3	1.3	75.9	77.8	68.5
Financials	−0.9	−0.9	−0.9	27.8	35.7	24.4
Health Care	−10.0	−7.8	−3.1	13.7	17.3	12.9
Industrials	−0.4	−0.7	−1.4	19.0	25.5	19.4
Information Technology	−6.0	−0.9	−0.7	30.9	31.4	17.7
Materials	−0.4	−0.8	−0.1	32.1	39.1	31.8
Real Estate	0.9	4.4	2.4	34.7	47.4	47.4
Utilities	−7.4	−6.9	−6.3	16.7	24.4	23.1
MSCI World	−3.0	−2.4	−1.7	25.5	31.5	25.0

Nevertheless, we observe a lot of discrepancies between sectors. While Utilities and Energy are the two major contributors to the MSCI World's carbon intensity, Utilities exhibits a negative carbon momentum, but Energy has a positive carbon momentum. We have also reported the share of each sector's constituents exhibiting positive carbon momentum. If we consider scope $\mathcal{SC}_{1-3}^{\text{up}}$, 68.5% of the companies belonging to the Energy sector have increased their carbon intensities these last years. This figure is 44.3% for Communication Services, and 47.4% for Real Estate. It is also interesting to notice that the Real Estate sector has a low-carbon allocation but a positive carbon momentum. Introducing a carbon momentum constraint is thus crucial in the optimization to avoid overweighting companies with positive carbon momentum.

Optimization problem Le Guenedal and Roncalli (2022) describe several mathematical approaches to formulating the portfolio decarbonization problem. We focus on the max-threshold solution since it is the most accepted method among professionals. Let x be a portfolio and Σ the covariance matrix of stock returns. The objective function is to minimize the tracking error variance of Portfolio x with respect to Benchmark b subject to a carbon reduction constraint:

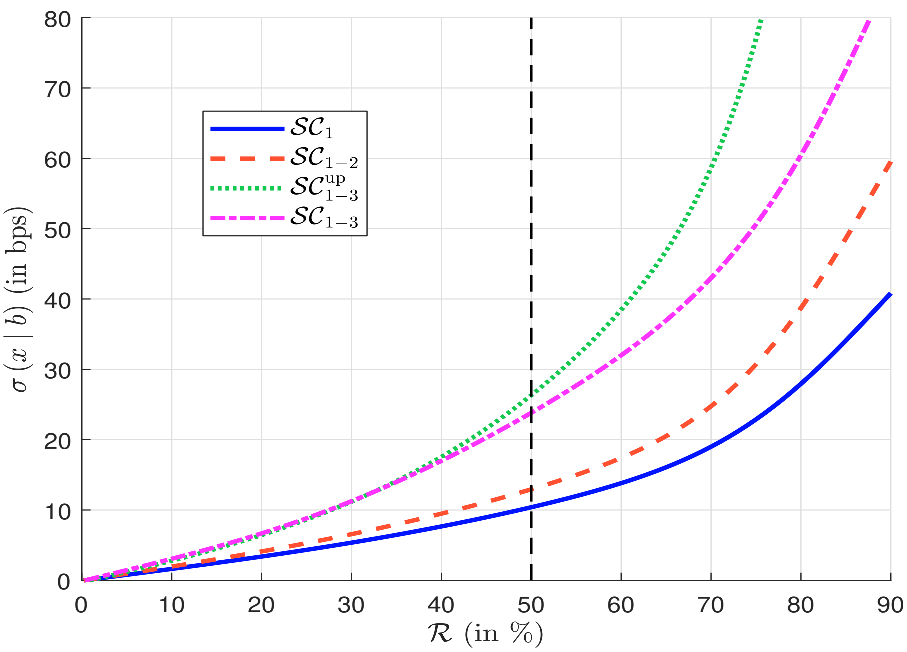
$$\begin{aligned} x^{\star}(\mathcal{R}) &= \arg\min \frac{1}{2}(x-b)^{\top}\Sigma(x-b) \\ &\text{s.t.} \left\{ \begin{array}{l} \mathcal{CI}(x) \leq (1-\mathcal{R}) \cdot \mathcal{CI}(b) \\ x \in \Omega_1 \cap \Omega_2 \end{array} \right. \end{aligned} \tag{61}$$

where $\mathcal{R}$ is the reduction rate and $\Omega = \Omega_1 \cap \Omega_2$ is a set of constraints. The first set $\Omega_1 = \left\{x : \mathbf{1}_n^{\top} x = 1, \mathbf{0}_n \leq x \leq \mathbf{1}_n\right\}$ implies that we obtain a long-only portfolio, whereas the second set Ω_2 controls the weight deviation between Portfolio x and Benchmark b. For instance, we can use $\Omega_2 = \left\{x : m_w^- b \leq x \leq m_w^+ b\right\}$ where $m_w^- \in [0,1[$ and $m_w^+ \in [1,\infty[$. In this case, the portfolio's weight x_i can only deviate from the benchmark's weight b_i by lower and upper ratios m_w^- and m_w^+. Typical figures are $m_w^- = 1/2$ and $m_w^+ = 2$. Another approach consists in controlling the sector deviations. In this case, we can use a relative deviation allowance — $\Omega_2 = \left\{\forall j : m_s^- \sum_{i \in \mathcal{S}ector_j} b_i \leq \sum_{i \in \mathcal{S}ector_j} x_i \leq m_s^+ \sum_{i \in \mathcal{S}ector_j} b_i\right\}$ — or an absolute deviation allowance — $\Omega_2 = \left\{\forall j : \left|\sum_{i \in \mathcal{S}ector_j} (x_i - b_i)\right| \leq \delta_s^+\right\}$. In what follows, we use 4 sets of constraints:

$\mathcal{C}_0$ only imposes long-only constraints, $\mathcal{C}_1\left(m_w^-, m_w^+\right)$ adds stock weight constraints, $\mathcal{C}_2\left(m_s\right)$ adds sector relative allocation constraints with $m_s^- = 1/m_s$ and $m_s^+ = m_s$, and $\mathcal{C}_3\left(m_w^-, m_w^+, m_s\right) = \mathcal{C}_1\left(m_w^-, m_w^+\right) \cap \mathcal{C}_2\left(m_s\right)$ combines $\mathcal{C}_1$ and $\mathcal{C}_2$.

Results We have reported the tracking error volatility (expressed in bps) in Figure 16 when we consider the $\mathcal{C}_0$ constraint. The tracking risk increases when we include scope 2 or upstream scope 3, whereas downstream scope 3 reduces it because of its large dispersion. If we now impose the classical weight constraint $\mathcal{C}_1\left(1/3, 3\right)$, which is very popular in indexing management, we observe a high increase in the tracking error volatility (Figure 17). Moreover, we generally have no solution for $\boldsymbol{\mathcal{R}} > 60\%$. The issue comes from the lower bound, which is way to narrow. Indeed, portfolio decarbonization is, above all, an exclusion process. By imposing a lower bound, we then limit portfolio decarbonization. For instance, we obtain similar results between constraint $\mathcal{C}_1\left(0, 3\right)$ and constraint $\mathcal{C}_0$. Nevertheless, we must be careful when choosing m_w^+, because a low value can lead to infeasible solutions. For instance, this is the case of constraint $\mathcal{C}_1\left(0, 1.25\right)$, as shown in Figure 17. If we compare Figures 17 and 18, we notice that the impact of sector constraints is less important than the impact of weight constraints. For instance, constraint $\mathcal{C}_2\left(1\right)$ imposes match the benchmark sectoral allocations. For low reduction rates (less than 50%), the increase of tracking risk is lower than 30 bps. The combination of weight and sectoral constraints is a more difficult exercise as shown in the bottom panels in Figure 18.

Figure 16: Impact of the carbon scope on the tracking error volatility (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)



Remark 18. *At first sight, it may be surprising that weight constraints are more binding than sectoral constraints. Indeed, we generally consider that the sector contribution is greater than the idiosyncratic contribution. Therefore, we expect that the inter-class dispersion largely dominates the intra-class variance. Nevertheless, this viewpoint is biased because it considers homogeneous sectors. In our case, we use level one of the GICS classification. The concept of sector is then very heterogeneous. Within a particular sector, we can have low-carbon and high-carbon issuers.*

Figure 17: Impact of the $\mathcal{C}_1$ constraint on the tracking error volatility (MSCI World, Jun. 2022)

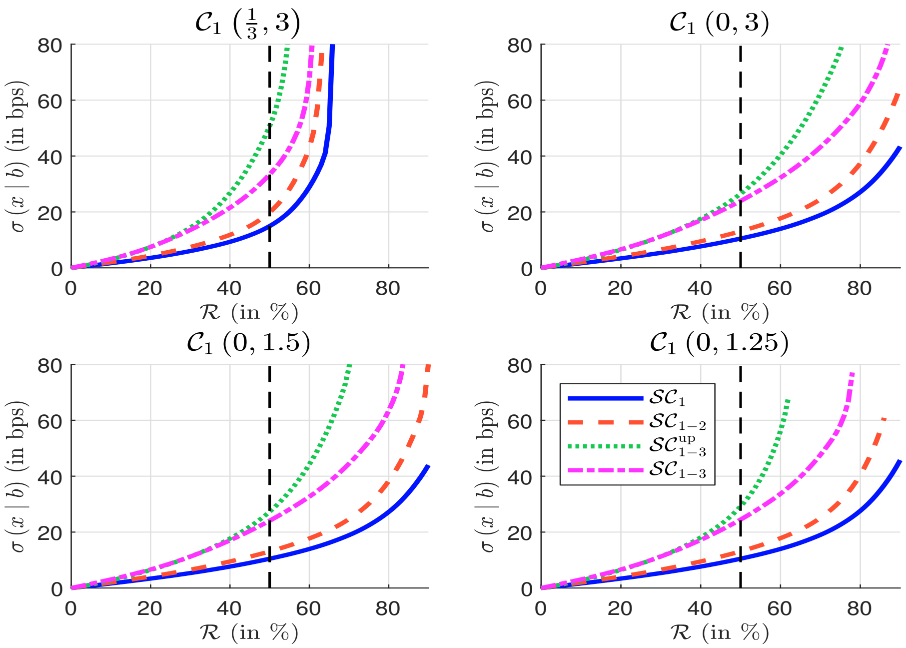
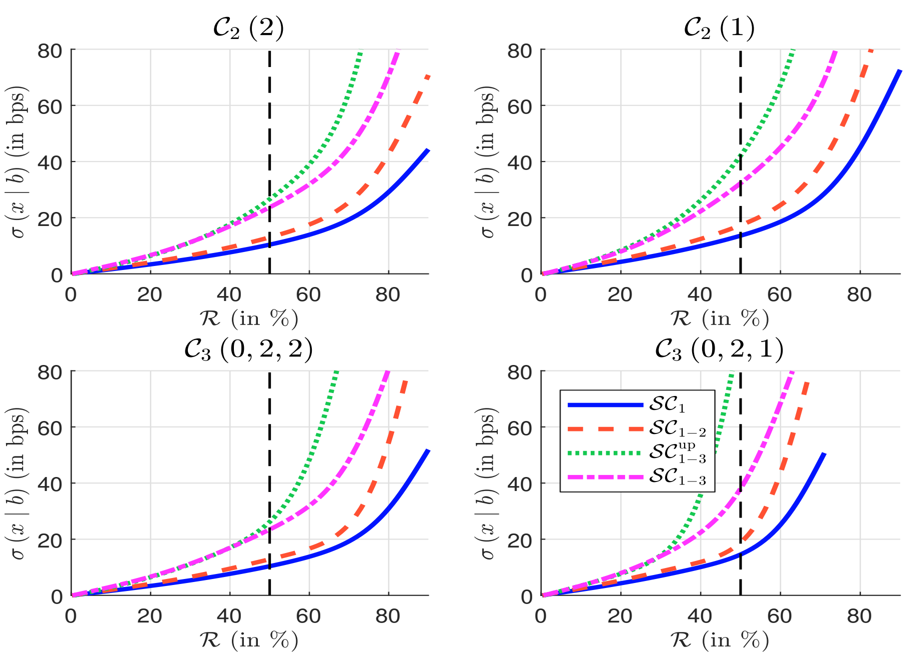


Figure 18: Impact of the $\mathcal{C}_2$ and $\mathcal{C}_3$ constraints on the tracking error volatility (MSCI World, Jun. 2022)



For instance, we have reported the boxplots of carbon intensity per sector in Figures 42 and 43 on page 122. We can easily find issuers with low and high carbon footprints for each sector. This is why portfolio decarbonization cannot be reduced to arbitrage between sectors.

Table 14: Sector allocation in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.95	8.15	8.42	8.78	9.34	10.13	12.27
Consumer Discretionary	10.56	10.69	10.69	10.65	10.52	10.23	9.62	6.74
Consumer Staples	7.80	7.80	7.69	7.48	7.11	6.35	5.03	1.77
Energy	4.99	4.14	3.65	3.10	2.45	1.50	0.49	0.00
Financials	13.56	14.53	15.17	15.94	16.90	18.39	20.55	28.62
Health Care	14.15	14.74	15.09	15.50	16.00	16.78	17.77	17.69
Industrials	9.90	9.28	9.01	8.71	8.36	7.79	7.21	6.03
Information Technology	21.08	21.68	22.03	22.39	22.88	23.51	24.12	24.02
Materials	4.28	3.78	3.46	3.06	2.56	1.85	1.14	0.24
Real Estate	2.90	3.12	3.27	3.41	3.57	3.72	3.71	2.51
Utilities	3.21	2.28	1.79	1.36	0.90	0.54	0.24	0.12

In Table 14, we have reported the sectoral allocation considering the $\mathcal{C}_0$ constraint. We observe that portfolio decarbonization is a strategy that is long on Financials and short on Energy, Materials and Utilities, although the extent of reallocation depends on the scope[35]. In particular, we notice that the most favorable case for the Financials sector is when we consider upstream scope 3. Moreover, we observe some strong non-linearities. The allocation in a given sector may increase when the reduction rate is low, but it may also strongly decrease when the reduction rate is very high[36]. These results are obtained with the $\mathcal{C}_0$ constraint, but can be generalized to $\mathcal{C}_1$ or $\mathcal{C}_2$ constraints. Indeed, by imposing sector neutrality for instance, we observe the same phenomenon but at a sub-level category, typically between industries or sub-industries.

Transition dimension As said previously, a decarbonization strategy does not necessarily support a transition to a low-carbon economy for two main reasons. The first one is that the resulting portfolio does not naturally allocate capital toward green activities, as illustrated in Table 15. The green intensity is defined as the green revenue share of the portfolio. We observe a decreasing function between the green intensity and the reduction level. This negative correlation between decarbonization and transition dimensions is particularly problematic from a dynamic perspective. Thus, it is necessary to introduce a green intensity constraint to prevent aligned portfolios from having a lower green intensity.

Similarly, we compute the carbon momentum $\boldsymbol{\mathcal{CM}}^{\mathcal{L}ong}$ of decarbonized portfolios[37]. Most of the time, we observe that the carbon momentum of the decarbonized portfolio is higher than the benchmark. Thus, if all companies pursue their past efforts, the benchmark will decarbonize itself faster than the optimized portfolio. In this scenario, the benchmark's future carbon intensity would be lower than the decarbonized portfolio's future carbon intensity.

[35] See Tables 45 and 46 on page 103.

[36] For example, this is the case of the Communication Discretionary sector when we consider scope $\boldsymbol{\mathcal{SC}}_{1-3}$.

[37] In the sequel, we use the $\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$ carbon momentum to perform all the analysis.

Table 15: Green intensity in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	5.24	5.21	5.19	5.18	5.16	5.12	5.08	5.01
$\mathcal{SC}_{1-2}$		5.17	5.14	5.09	4.99	4.83	4.64	4.52
$\mathcal{SC}_{1-3}^{\mathrm{up}}$		5.15	5.07	4.89	4.69	4.42	3.90	0.68
$\mathcal{SC}_{1-3}$		5.17	5.12	5.05	4.97	4.80	4.55	3.73

Table 16: $\mathcal{SC}_{1-3}^{\mathrm{up}}$ carbon momentum in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	−1.7	−1.5	−1.3	−1.2	−1.2	−1.3	−1.6	−1.8
$\mathcal{SC}_{1-2}$		−1.5	−1.3	−1.3	−1.4	−1.7	−1.9	−2.6
$\mathcal{SC}_{1-3}^{\mathrm{up}}$		−1.7	−1.7	−1.8	−2.1	−2.8	−4.5	−7.7
$\mathcal{SC}_{1-3}$		−1.8	−1.8	−1.7	−1.6	−1.8	−1.8	−1.8

6.1.2 Bond portfolios

Benchmark analysis We report the carbon intensity of the Global Corp. index[38] and its GICS sectors[39] in Table 17. The index carbon intensity is 249 tCO_2e/\$ mn for scope 1, 286 tCO_2e/\$ mn if we include scope 2, 435 tCO_2e/\$ mn if we add upstream scope 3, and finally 1265 tCO_2e/\$ mn if we consider the full scope 3. As in the equity case, we notice a factor of 3 between the full scope 3 and the upstream scope 3. We also observe the same high discrepancy between sectors and hence the same impact on portfolio decarbonization.

Table 17: Carbon intensity in tCO_2e/\$ mn per GICS sector (Global Corp., June 2022)

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	4	28	270	309
Consumer Discretionary	22	73	242	1 011
Consumer Staples	36	65	485	700
Energy	610	698	997	5 694
Financials	1	7	33	590
Health Care	10	21	115	144
Industrials	143	165	318	1 390
Information Technology	11	34	119	254
Materials	655	835	1 167	2 347
Real Estate	25	107	149	904
Utilities	1 666	1 750	2 031	2 957
Global Corp.	249	286	435	1 265

[38] Only 89% of the index has carbon data since private/unlisted issuers are not covered by Trucost. For these issuers, we associate the average weighted carbon data of their related GICS sector.

[39] These sectors are usually used in the equity space. Therefore, we perform a mapping from the Merrill Lynch sectors to have a comparable sector view.

Table 18: Sectoral contribution in % (Global Corp., June 2022)

Sector	Index	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	7.34	0.12	0.73	4.55	1.79
Consumer Discretionary	5.97	0.53	1.52	3.32	4.77
Consumer Staples	6.04	0.88	1.38	6.74	3.34
Energy	6.49	15.88	15.82	14.88	29.20
Financials	33.91	0.15	0.84	2.58	15.81
Health Care	7.50	0.30	0.56	1.99	0.85
Industrials	8.92	5.13	5.14	6.52	9.80
Information Technology	5.57	0.23	0.65	1.53	1.12
Materials	3.44	9.04	10.05	9.24	6.39
Real Estate	4.76	0.48	1.78	1.64	3.40
Utilities	10.06	67.25	61.52	47.01	23.52

In Table 18, we report the contribution of each sector to the portfolio carbon intensity. We notice that with a different sector allocation than the MSCI World, Energy, Materials, and Utilities sectors are still the main contributors to carbon intensity. These sectors also exhibit the highest ratios of risk contribution in the benchmark, whereas Financials, Health Care, and Information Technology are the sectors with the lowest ratios.

Remark 19. *The corporate bond index structure is significantly different from the equity index structure because of the weight of the Financials sector. Therefore, the results we have obtained for equity portfolios might not be valid for bond portfolios.*

Optimization problem To replicate a market index, fund managers may hold the same securities or a stratified sampling of the securities that comprise the index (Neyman, 1934). Therefore, they track the index portfolio by exhibiting the same risk/return characteristics. In the fixed income space, modified duration (MD) and duration-times-spread (DTS) are the most widely used risk metrics[40]. Indeed, historical volatility, which measures the risk of equity portfolios, is not a reliable predictor of bond volatility since bonds are less frequently traded and mature over time.

In the case of bonds, the objective function is to minimize sectoral active credit risk and the active share (AS) of Portfolio x with respect to Benchmark b subject to a carbon reduction constraint[41]:

$$\begin{aligned} x^{\star}(\boldsymbol{\mathcal{R}}) &= \arg\min \varphi \underbrace{\sum_{s=1}^{n_{\mathcal{S}ector}} \left| \sum_{i \in s} (x_i - b_i) \cdot \mathrm{DTS}_i \right|}_{\text{DTS component}} + \underbrace{\frac{1}{2} \sum_{i \in b} |x_i - b_i|}_{\text{AS component}} \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}(x) \leq (1 - \boldsymbol{\mathcal{R}}) \cdot \boldsymbol{\mathcal{CI}}(b) \\ x \in \Omega_1 \cap \Omega_2 \end{array} \right. \end{aligned} \tag{62}$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate, $\Omega_1 \cap \Omega_2$ is a set of constraints and φ is the trade-off coefficient between DTS and AS components[42]. As in the case of equities, the first set $\Omega_1 = \left\{ x : \mathbf{1}_n^\top x = 1, \mathbf{0}_n \leq x \leq \mathbf{1}_n \right\}$

[40] MD is the sensitivity of the bond return to interest risk, and DTS measures the systematic exposure to credit risk by quantifying sensitivity to a shift in the yield spread (Ben Dor *et al.*, 2007).

[41] The current exercise does not consider minimum tradable, lot size or the liquidity of bonds. Therefore, solutions may exist theoretically, but their implementation may be challenging.

[42] φ is set to 50, implying that the trade-off is 1% of active share for 2 bps of DTS.

implies that we obtain a long-only portfolio, whereas the second set Ω_2 controls the risk metrics deviation between Portfolio x and Benchmark b. We can use $\Omega_2 = \Omega_{2'} \cap \Omega_{2''} \cap \Omega_{2'''}$ where:

$$\Omega_{2'} = \left\{ x : \sum_{i=1}^{n} (x_i - b_i) \cdot \mathrm{MD}_i = 0 \right\}$$

,

$$\Omega_{2''} = \left\{ x : \forall j, \sum_{i \in \mathcal{B}ucket(j)} (x_i - b_i) = 0 \right\}$$

and

$$\Omega_{2'''} = \left\{ x : \forall j, \sum_{i \in \mathcal{R}ating(j)} (x_i - b_i) = 0 \right\}$$

The $\Omega_{2'}$ constraint neutralizes the modified duration at the portfolio level, whereas $\Omega_{2''}$ and $\Omega_{2'''}$ constraint the portfolio to have the same weights as the benchmark per maturity bucket[43] and rating category[44]. We choose not to add further constraints because the current problem is already highly constrained at the sector level, and therefore no sector will vanish when a solution is found.

Results We have reported in Figures 19 and 20 the duration-times-spread tracking risk:

$$\mathrm{DTS}\left(x \mid b\right) = \sum_{s=1}^{n_{Sector}} \left| \sum_{i \in s} (x_i - b_i) \cdot \underset{i}{\mathrm{DTS}} \right|$$

and the active share:

$$\mathrm{AS}\left(x \mid b\right) = \frac{1}{2} \sum_{i \in b} \left| x_i - b_i \right|$$

We observe that the tracking risk is low when we consider the DTS component, whereas it is significant when we focus on the weight component. In particular, $\mathrm{AS}\left(x \mid b\right)$ increases when we include upstream and downstream scope 3. On average, there is a factor of two between $\boldsymbol{\mathcal{SC}}_{1-3}$ and $\boldsymbol{\mathcal{SC}}_{1-2}$. Moreover, we notice that the active share accelerates where the reduction rate $\mathcal{R}$ is above 85% and can reach 50%.

Table 19 shows the deviation of sectoral allocation versus the benchmark when considering the $\boldsymbol{\mathcal{SC}}_{1-3}$ scope. We observe that the decarbonization process is also a strategy that is long on Financials and short on Materials and Utilities. As shown in Tables 47–49 on page 104, reallocation depends on the scope. Health care, Communication Services, Consumer Discretionary, and Information Technology weights are very close to their benchmark's. Regarding the other sectors, the strategy may point in contradictory directions according to the scope. For instance, it is short on Energy with $\boldsymbol{\mathcal{SC}}_{1-3}$ but long on Energy with $\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$. Likewise, it is long on Industrials with scope $\boldsymbol{\mathcal{SC}}_{1-3}$, but no conclusion can be drawn regarding other scopes.

Table 20 shows that the yield of the decarbonized portfolio is lower and decreases with the reduction rate. This yield difference in the full scope is due to the lower contribution of the Energy, Materials, and Utilities sectors, partially offset by the higher contribution of Financials and Industrials (see Table 50 on page 106). The breakdown by ratings and durations suggests that BBB-rated bonds and bonds whose duration is between two and five years explain the lower yield.

Remark 20. *In Tables 51 and 52 on page 107, we focus on the two main benchmark sectors: Financials and Utilities. We note that the higher contribution for Financials comes mainly from the*

[43] We use the following buckets: 0Y–2Y, 2Y–5Y, 5Y–7Y, 7Y–10Y and 10Y+.
[44] The rating categories are AAA–AA, A and BBB.

Figure 19: Impact of the carbon scope on the duration-times-spread in bps (Global Corp., Jun. 2022)

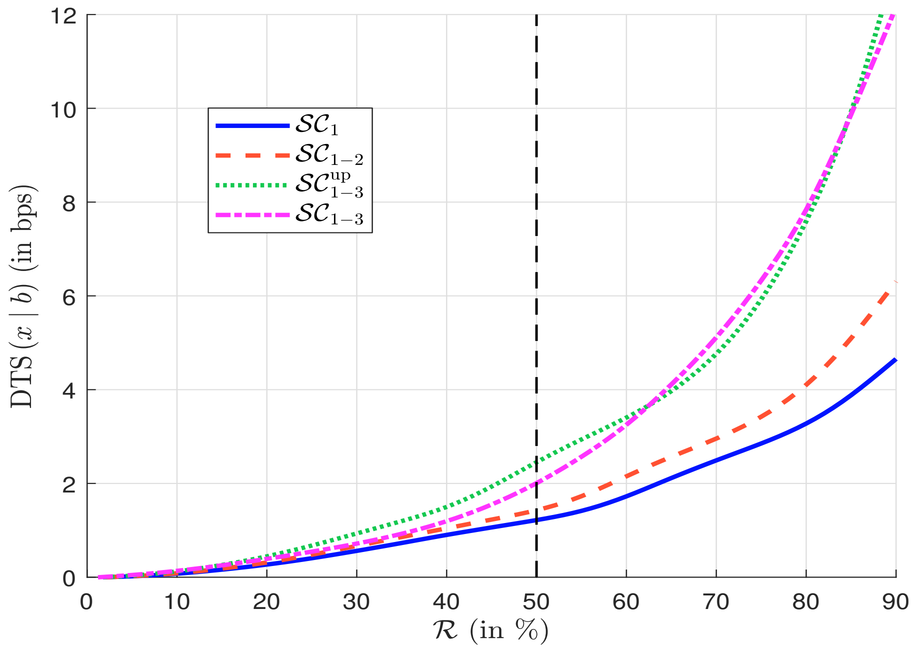


Figure 20: Impact of the carbon scope on the active share in % (Global Corp., Jun. 2022)

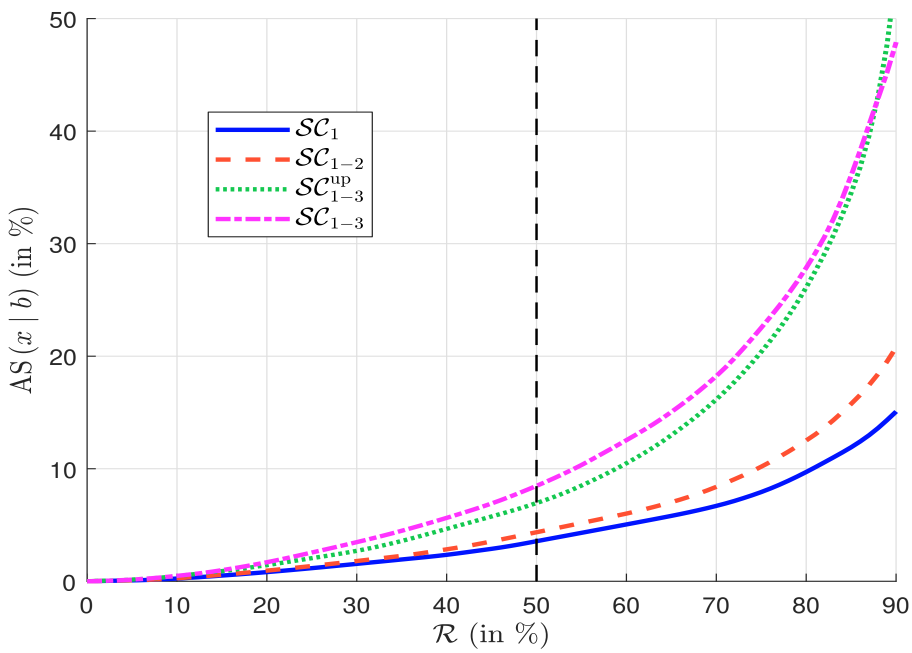


Table 19: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	0.01	0.00	0.03	0.09	0.09	−0.03	−0.04
Consumer Discretionary	5.97	0.00	−0.01	−0.03	−0.04	−0.51	−1.49	−2.42
Consumer Staples	6.04	0.00	0.00	0.00	0.00	−0.02	−0.65	−1.98
Energy	6.49	−1.00	−2.07	−2.65	−2.80	−3.26	−3.91	−3.97
Financials	33.91	0.73	1.75	2.05	2.18	3.45	4.95	5.09
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	0.02	−0.02
Industrials	8.92	0.46	0.70	1.27	2.42	3.15	4.63	9.21
Information Technology	5.57	0.00	0.02	0.02	0.03	0.03	−0.05	−0.30
Materials	3.44	−0.01	−0.13	−0.26	−0.32	−0.80	−1.19	−1.58
Real Estate	4.76	−0.02	−0.02	−0.02	−0.02	−0.10	−0.15	−0.83
Utilities	10.06	−0.17	−0.24	−0.42	−1.54	−2.02	−2.14	−3.18

Table 20: Yield variation in bps (Global Corp., Jun. 2022)

Scope	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
$\boldsymbol{\mathcal{SC}}_{1}$	422	−2	−2	−1	−6	−6	−8	−11
$\boldsymbol{\mathcal{SC}}_{1-2}$		−1	−2	−3	−3	−3	−10	−15
$\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$		−3	−3	−4	−10	−16	−23	−57
$\boldsymbol{\mathcal{SC}}_{1-3}$		0	−2	−3	−7	−8	−9	−22

short-duration overweighting (0Y–5Y of AAA–AA, 2Y–7Y of A, and the liquidity bucket of BBB). The optimizer also underweights BBB-rated bonds whose duration exceeds two years, resulting in restrained lower yields. In the meantime, regarding Utilities, the optimizer has progressively underweighted BBB-rated bonds and the 0Y–7Y bucket of A-rated bonds. The outcome is partially reallocated to overweight the high-duration of A-rated bonds.

Transition dimension In Table 21, we see that relative to the benchmark, the decarbonized portfolio has better green intensity[45] that increases with the reduction rate. However, this finding does not apply to scope $\mathcal{SC}_{1-2}$. On the other hand, the green intensity never exceeds twice the benchmark green intensity[46].

Table 21: Green intensity in % (Global Corp., Jun. 2022)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	3.49	3.69	3.84	3.99	4.30	4.70	5.27	5.97
$\mathcal{SC}_{1-2}$		3.44	3.39	3.40	3.42	3.44	3.45	3.06
$\mathcal{SC}_{1-3}^{\text{up}}$		3.55	3.53	3.85	3.95	3.94	3.39	2.00
$\mathcal{SC}_{1-3}$		3.57	3.74	3.97	4.74	5.21	5.84	5.59

We illustrate in Table 22 the carbon momentum of the decarbonized portfolio. As its reference, the decarbonized portfolio exhibits negative carbon intensity trends. We note that the carbon momentum of the decarbonized portfolio is generally above the benchmark. Therefore, imposing a constraint on the carbon momentum may help the aligned portfolio to decarbonize faster than the benchmark.

Table 22: Carbon momentum in % (Global Corp., Jun. 2022)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	−2.93	−2.29	−1.92	−1.71	−1.26	−1.11	−1.28	−0.93
$\mathcal{SC}_{1-2}$		−2.27	−2.01	−1.89	−1.45	−1.89	−2.30	−2.07
$\mathcal{SC}_{1-3}^{\text{up}}$		−2.27	−2.03	−1.85	−2.26	−2.74	−3.14	−5.27
$\mathcal{SC}_{1-3}$		−3.06	−3.14	−3.12	−1.99	−1.78	−1.97	−0.98

6.2 Integrated approach

The previous analysis has shown that portfolio decarbonization recovers only one dimension of net zero investing: The reduction of the carbon footprint of asset portfolios. We now consider extending the previous approach by adapting the mathematical optimization problem. This approach is integrated because it tries to solve the problem in one step by integrating the transition dimension, which is multi-faceted.

[45] 6.42% of the benchmark has no green data. We apply a zero green intensity for the related issuers.

[46] Table 53 on page 109 displays the results when we apply the average weighted green intensity per sector to issuers with no green data. The results are consistent with the above findings.

6.2.1 Equity portfolios

Dynamic decarbonization While the decarbonization problem finds an optimal portfolio $x^{\star}(\boldsymbol{\mathcal{R}})$ with respect to a given reduction rate $\boldsymbol{\mathcal{R}}$, the alignment problem defines an optimal portfolio $x^{\star}(t)$ with respect to a given date t. Therefore, this second problem can be seen as a special case of the first problem, where we use the mapping function between the date t and the reduction rate $\boldsymbol{\mathcal{R}}$ (Le Guenedal and Roncalli, 2022). In this case, the decarbonization problem becomes dynamic:

$$\begin{aligned} x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\ &\text{s.t.} \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \\ x \in \Omega_1 \cap \Omega_2(t) \end{array} \right. \end{aligned} \tag{63}$$

where t_0 is the base year and $\boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right)$ is the carbon intensity of the benchmark at time t_0. We notice that the benchmark $b(t)$, the covariance matrix $\Sigma(t)$, the carbon intensity $\boldsymbol{\mathcal{CI}}(t, x)$ and the set of additional constraints $\Omega_2(t)$ are functions of time t. This means that the data are updated every time we rebalance the portfolio[47]. In this framework, the constraint $\boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right)$ corresponds to the net zero emissions scenario, which is expressed in terms of carbon intensity. We have the following properties:

- The decarbonization of the aligned portfolio becomes easier with time if the benchmark decarbonizes itself:
$$\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) \ll \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \qquad \text{for } t > t_0 \tag{64}$$
- The decarbonization of the aligned portfolio becomes trickier with the time if the benchmark carbonizes itself:
$$\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) \gg \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \qquad \text{for } t > t_0 \tag{65}$$
- The aligned portfolio corresponds to the benchmark portfolio if the decarbonization of the benchmark is sufficiently strong:
$$\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \tag{66}$$

Since we have $\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) = \sum_{i=1}^{n} \boldsymbol{\mathcal{CI}}_i(t) \cdot b_i(t)$, the decarbonization part of a net zero investing process is highly influenced by two pictures: changes in the benchmark weights and the carbon intensity of the assets. Indeed, we can imagine that the decarbonization process becomes easier over time, because the market capitalization of green assets grows faster than the market capitalization of brown assets and/or because the global decarbonization of the world is well established and follows the right way.

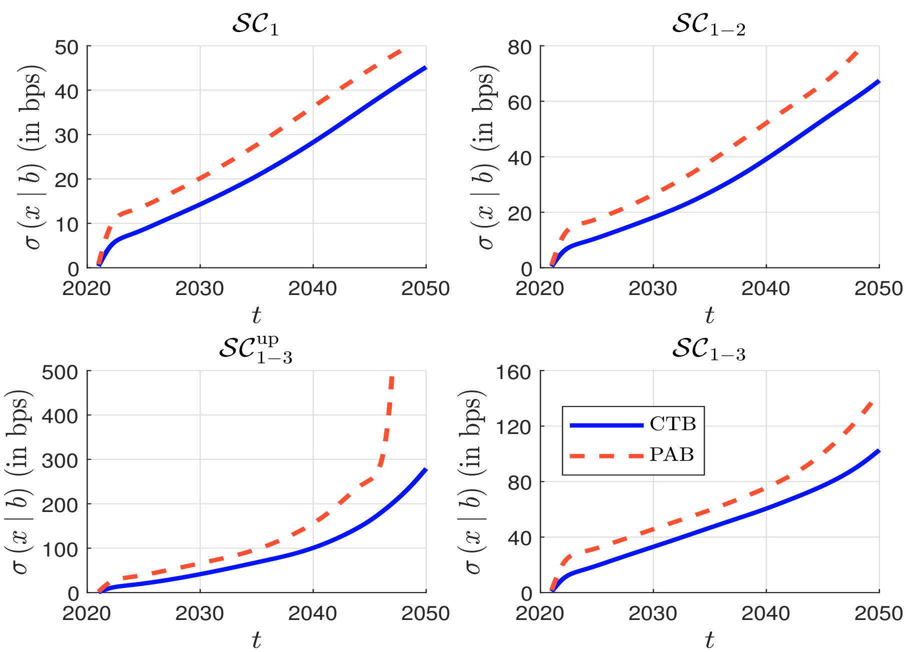
Remark 21. *In what follows, we consider that the data are not updated since we cannot guess or predict the benchmark composition in the future, the evolution of the covariance matrix, the level of carbon intensity, etc. As in Le Guenedal and Roncalli (2022), we assume that the world does not change. Of course, this is not realistic, but we are more interested in an order of magnitude of the tracking risks and a comparison between the different approaches rather than determining the optimal solutions.*

In Figure 21, we show the relationship between the time and the tracking error volatility with respect to the scope when considering the CTB and PAB decarbonization pathways. As observed by Le Guenedal and Roncalli (2022), including scope 3 has a significant impact on tracking risk, especially when considering the upstream scope 3. On average, including scope 3 results in

[47] For instance, at time $t+1$, the optimization problem depends on the data available at this current date and not at the past date t.

multiplying the tracking risk by a factor of three. If we include weight and sector constraints, we may face situations where we do not find a solution (Figure 22). This is particularly true when imposing sectoral neutrality. In this case, the solution may not exist even before 2030 for the PAB decarbonization pathway. In order to have acceptable solutions, we relax these constraints and choose the $\mathcal{C}_3(0, 10, 2)$ configuration to challenge the $\mathcal{C}_0$ case (Figure 23).

Figure 21: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)



Controlling the greenness As explained above, we must introduce the transition dimension. The PAB framework defines the concept of high climate impact sectors (HCIS). It lists several strategic sectors with respect to NACE European classification and imposes the following transition constraint:

$$\mathcal{H}_{CIS}(x(t)) \geq \mathcal{H}_{CIS}(b(t)) \tag{67}$$

where $\mathcal{H}_{CIS}(x) = \sum_{i \in \mathcal{H}_{CIS}} x_i$ is the weight of the portfolio that falls into HCIS sectors. As demonstrated by Le Guenedal and Roncalli (2022), this constraint has little impact on the transition dimension. Indeed, it does not help to maintain exposure in key sectors. Moreover, we can show that it does not help finance the transition to a low-carbon economy. This is why it is better to use a green intensity measure instead. We obtain the following optimization problem:

$$\begin{aligned} x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\ &\text{s.t.} \left\{ \begin{array}{ll} \boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ \boldsymbol{\mathcal{GI}}(t, x) \geq \left(1 + \boldsymbol{\mathcal{G}}(t)\right) \cdot \boldsymbol{\mathcal{GI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{Transition} \\ x \in \Omega_1 \cap \Omega_2(t) & \end{array} \right. \end{aligned} \tag{68}$$

Concerning the transition dimension, we can use the current benchmark as the anchor point and define an increasing function for the greenness multiplier $\boldsymbol{\mathcal{G}}(t)$. Another solution is to replace this

Figure 22: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 2, 1)$ constraint)

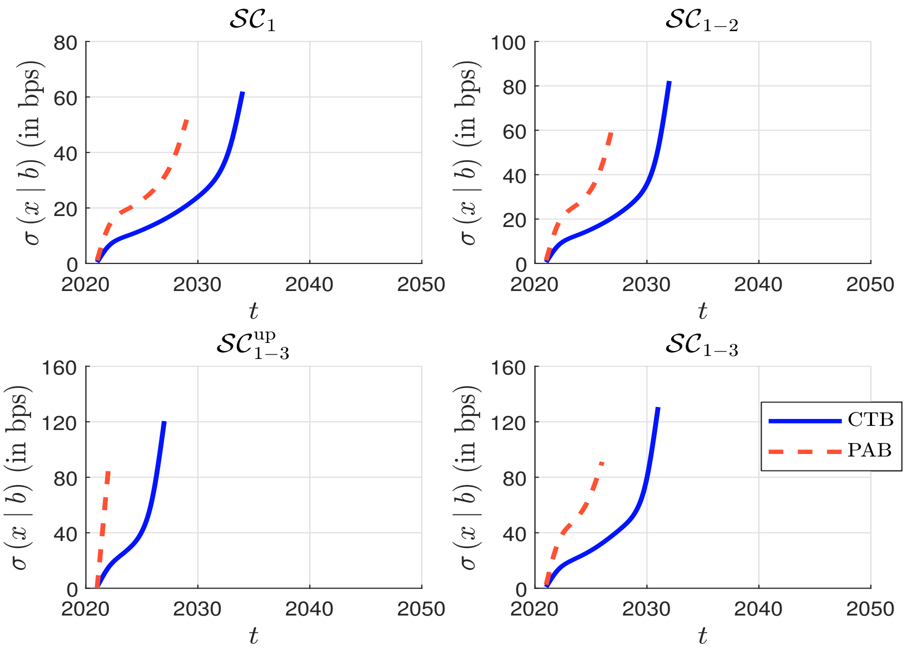
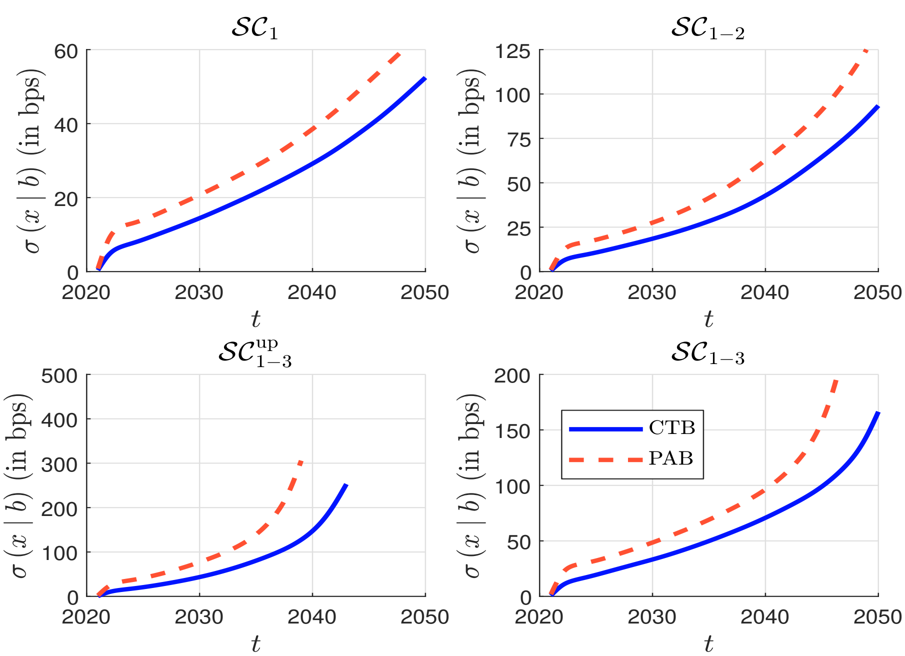


Figure 23: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint)



constraint by the following one:

$$\mathcal{GI}(t,x) \geq (1+\mathcal{G}) \cdot \mathcal{GI}\left(t, b(t)\right) \tag{69}$$

The underlying idea is to maintain a green intensity for the net zero portfolio that is higher than the green intensity of the benchmark.

Table 23: Additional tracking error cost in bps of the greenness constraint (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{G} = 0\%$				
$\mathcal{SC}_1$	0	0	0	0	0	0	0	0	0
$\mathcal{SC}_{1-2}$	0	0	0	0	0	0	0	0	1
$\mathcal{SC}_{1-3}^{\text{up}}$	0	0	0	0	0	1	5	27	
$\mathcal{SC}_{1-3}$	0	0	0	0	0	0	0	2	6
					$\mathcal{G} = 100\%$				
$\mathcal{SC}_1$	22	21	21	20	17	15	13	11	11
$\mathcal{SC}_{1-2}$	21	20	20	19	18	17	16	17	19
$\mathcal{SC}_{1-3}^{\text{up}}$	17	16	15	15	14	19	40	106	
$\mathcal{SC}_{1-3}$	16	15	14	14	12	12	13	22	43
					$\mathcal{G} = 200\%$				
$\mathcal{SC}_1$	51	51	50	49	45	42	38	35	33
$\mathcal{SC}_{1-2}$	50	50	49	48	46	45	43	48	54
$\mathcal{SC}_{1-3}^{\text{up}}$	44	43	42	41	39	50	95	257	
$\mathcal{SC}_{1-3}$	43	42	41	40	36	34	39	57	112

We have implemented a fixed greenness multiplier $\mathcal{G}$. In Table 23, we report the additional tracking error cost due to the transition constraint when we consider the PAB decarbonization pathway. We notice that this cost is equal to zero or relatively negligible when the greenness of the benchmark is to be maintained ($\mathcal{G} = 0\%$). Nevertheless, this constraint leads to a portfolio with a green intensity of only 5.24%, which may be weak for a net zero investor who wants to finance the transition. Doubling the green intensity ($\mathcal{G} = 100\%$) implies a marginal tracking error cost between 10 and 20 bps most of the time, except for the scope 3 and long time horizon. We also observe that the relationship between the green intensity and the tracking error cost is highly non-linear. Indeed, if we target a green intensity of 15%, which corresponds to a greenness multiplier $\mathcal{G}$ of about 200%, the additional cost lies between 35 and 100 bps.

Remark 22. *If we consider the $\mathcal{C}_3(0,10,2)$ constraint, we observe an increase in the tracking error which is relatively low until 2030 if $\mathcal{G} \leq 100\%$ (see Table 54 on page 110). Moreover, it becomes more and more difficult to find a solution when the greenness multiplier is equal to 200%.*

Managing the carbon momentum In order to manage the carbon momentum, we add a new constraint:

$$\begin{aligned} x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\ &\text{s.t.} \left\{ \begin{array}{ll} \mathcal{CI}(t,x) \leq \left(1 - \mathcal{R}(t_0,t)\right) \cdot \mathcal{CI}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ x \in \Omega_1 \cap \Omega_2(t) & \\ x \in \Omega_3(t) & \longleftarrow \text{Momentum} \end{array} \right. \end{aligned} \tag{70}$$

For instance, we can impose that the carbon momentum of the portfolio is lower than a global threshold:

$$\Omega_3(t) = \left\{ x : \mathcal{CM}^{\mathcal{L}ong}(t, x) \leq \mathcal{CM}^{\star} \right\} \tag{71}$$

In this case, the optimization program will overweight assets with negative momentum. For instance, if $\mathcal{CM}^{\star}$ is set to -7%, we expect the aligned portfolio to decarbonize itself by 7%. However, the previous constraint does not preclude the inclusion, or the overweighting, of companies with rising carbon intensities. Another approach consists in implementing an exclusion process:

$$\Omega_3(t) = \left\{ \mathcal{CM}_i^{\mathcal{L}ong}(t) \geq \mathcal{CM}^{+} \Rightarrow x_i = 0 \right\} \tag{72}$$

where $\mathcal{CM}^{+}$ is an acceptable upper bound. For example, if $\mathcal{CM}^{+}$ is set to 0, we exclude all the issuers presenting a positive carbon momentum.

Remark 23. *Another approach consists in imposing higher self-decarbonization than the benchmark:*

$$\Omega_3(t) = \left\{ x : \mathcal{CM}^{\mathcal{L}ong}(t, x) \leq \mathcal{CM}^{\mathcal{L}ong}\left(t, b(t)\right) - \Delta\mathcal{CM}^{\mathcal{L}ong}\left(x \mid b(t)\right) \right\} \tag{73}$$

This is equivalent to the global threshold approach where:

$$\mathcal{CM}^{\star} = \mathcal{CM}^{\mathcal{L}ong}\left(t, b(t)\right) - \Delta\mathcal{CM}^{\mathcal{L}ong}\left(x \mid b(t)\right) \tag{74}$$

For instance, we saw in Table 16 on page 63 that the carbon momentum of the MSCI World index is estimated at -1.7%. If we would like to improve the carbon momentum of the alignment portfolio, we can set $\mathcal{CM}^{\star} = -5\%$ or $\Delta\mathcal{CM}^{\mathcal{L}ong}\left(x \mid b(t)\right) = 3.3\%$.

Table 24 provides the marginal tracking error cost of adding a global momentum constraint to the $\mathcal{C}_0$ optimization problem. If $\mathcal{CM}^{\star} = -5\%$, the cost is lower than 10 bps, and decreases with the year. If $\mathcal{CM}^{\star} = -7\%$, we can observe a cost greater than 10 bps before 2030. Contrary to the green intensity, the weight constraint $\mathcal{C}_3(0, 10, 2)$ has a significant impact. Indeed, the cost is multiplied by a factor of two at the beginning of the period[48].

Table 24: Additional tracking error cost in bps of a global momentum threshold approach (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	9	9	9	9	7	5	3	2	2
$\mathcal{SC}_{1-2}$	8	7	7	7	5	3	1	1	0
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	4	3	2	2	0	0	0	0	
$\mathcal{SC}_{1-3}$	3	3	3	3	2	1	1	1	2
	$\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	17	17	16	16	13	10	8	6	4
$\mathcal{SC}_{1-2}$	15	15	14	13	10	7	4	2	1
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	8	7	6	5	1	0	0	0	
$\mathcal{SC}_{1-3}$	8	8	7	6	4	3	2	2	4

Let us now consider the exclusion approach. In Table 25, we give some statistics about the distribution of the carbon momentum[49]. It follows that 25% of issuers have a positive carbon

[48] See Table 55 on page 111.

[49] We remind that we use $\mathcal{SC}_{1-3}^{\mathrm{up}}$ for estimating the trend and at least 5 years of historical data. This explains that the carbon momentum does not cover 100% of the investment universe.

Table 25: Statistics of the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ (MSCI World, Jun. 2022)

Statistic	Median	Negative	Positive	$\mathcal{CM}_i <$		$\mathcal{CM}_i >$	
				-10%	-5%	$+5\%$	$+10\%$
Frequency (in %)	-1.5	75.1	24.9	5.9	14.0	2.3	0.8
Weight (in %)		72.8	24.6	4.3	12.2	1.0	0.5

momentum. If we consider the case $\mathcal{CM}_i > 5\%$, this figure is equal to 2.3% in terms of issuers and 1.0% in terms of allocation. Therefore, we expect that using an upper bound $\mathcal{CM}^+$ greater than 5% has little impact. Let us first consider the case $\mathcal{CM}^+ = 0\%$, implying that we exclude all the issuers with positive carbon momentum. Table 26 shows that the marginal tracking error cost is very high, especially at the beginning of the period. For example, the additional tracking error is greater than 100 bps until 2025. The reason is that a large proportion of issuers in the MSCI World index have a positive trend in their carbon intensity. Nevertheless, if we consider a higher value of $\mathcal{CM}^+$, the cost may be negligible. For instance, this is the case when $\mathcal{CM}^+$ is equal to 10%. Moreover, these different results remain valid with the $\mathcal{C}_3(0, 10, 2)$ constraint, as shown in Table 56 on page 111.

Table 26: Additional tracking error cost in bps of a momentum exclusion approach (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{CM}^+ = 0\%$				
$\mathcal{SC}_1$	123	122	121	120	114	107	100	93	88
$\mathcal{SC}_{1-2}$	121	119	118	117	109	98	87	78	66
$\mathcal{SC}_{1-3}^{\text{up}}$	109	105	102	98	80	63	37	10	
$\mathcal{SC}_{1-3}$	111	108	106	104	94	85	77	67	50
					$\mathcal{CM}^+ = 5\%$				
$\mathcal{SC}_1$	3	3	3	3	2	1	1	1	1
$\mathcal{SC}_{1-2}$	2	2	2	2	1	1	0	0	0
$\mathcal{SC}_{1-3}^{\text{up}}$	1	1	1	1	0	0	0	0	
$\mathcal{SC}_{1-3}$	2	2	1	1	1	1	1	0	0

Combining decarbonization and transition Finally, we combine all the constraints to define the final optimization problem. We consider the threshold approach for the carbon momentum and obtain:

$$x^\star(t) = \arg\min \frac{1}{2}\left(x - b(t)\right)^\top \Sigma(t)\left(x - b(t)\right) \tag{75}$$
$$\text{s.t.} \quad \begin{cases} \mathcal{CI}(t, x) \leq \left(1 - \mathcal{R}(t_0, t)\right) \cdot \mathcal{CI}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ x \in \Omega_{\mathcal{T}ransition}(t) & \longleftarrow \text{Transition} \\ x \in \Omega_1 \cap \Omega_2(t) \end{cases}$$

where the decarbonization dimension is defined by using the usual constraint $\mathcal{CI}(t, x) \leq \left(1 - \mathcal{R}(t_0, t)\right) \cdot \mathcal{CI}\left(t_0, b(t_0)\right)$ and the transition dimension is specified by the set of constraints $\Omega_{\mathcal{T}ransition}(t)$. In a first time, we assume that:

$$x \in \Omega_{\mathcal{T}ransition}(t) \Leftrightarrow \left\{ \begin{array}{ll} \mathcal{GI}(t,x) \geq \left(1+\mathcal{G}(t)\right) \cdot \mathcal{GI}\left(t_0, b(t_0)\right) & \longleftarrow \quad \text{Greenness} \\ \mathcal{CM}^{\mathcal{L}ong}(t,x) \leq \mathcal{CM}^{\star} & \longleftarrow \quad \text{Momentum} \end{array} \right. \tag{76}$$

For both CTB and PAB pathways, we consider the previous optimization program (95–96) and compute the solution using several sets of parameters: $\mathcal{C}_0$ vs. $\mathcal{C}_3(0,10,2)$, $\mathcal{G}=100\%$ vs. $\mathcal{G}=200\%$ and $\mathcal{CM}^{\star}=-5\%$ vs. $\mathcal{CM}^{\star}=-7\%$. The impact on the tracking error volatility and the decomposition between decarbonization and transition dimensions are reported in Figures 44–51 on pages 124–128. The results of these simulations clearly show that the transition dimension induces a significant cost. On average, if we focus on the case $\mathcal{G}=100\%$, $\mathcal{CM}^{\star}=-5\%$ and the PAB pathway, we observe that the additional tracking error cost for the years 2022–2030 is respectively equal to 23, 22, 16 and 15 bps for scopes $\mathcal{SC}_1$, $\mathcal{SC}_{1-2}$, $\mathcal{SC}_{1-3}^{\text{up}}$ and $\mathcal{SC}_{1-3}$ when we do not consider weight and sector constraints (Figure 24). These figures become 27, 25, 21 and 19 bps if we use the $\mathcal{C}_3(0,10,2)$ constraint (Figure 24). Moreover, there may not be a solution to the optimization problem through 2050, especially when the carbon footprint is based on upstream/downstream scope 3 emissions. Of course, all these results are very sensitive to the choice of the green multiplier $\mathcal{G}$ and the carbon threshold $\mathcal{CM}^{\star}$ as illustrated in Figures 44–51 on pages 124–128.

Remark 24. *The magnitude of the cost of combining green intensity and carbon momentum constraints is significantly higher than the cost of each constraint. This means that the two sub-dimensions of the transition pillar are not currently correlated. For instance, we have reported the scatter plot between the green intensity $\mathcal{GI}_i$ and the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ in Figure 26 and we do not observe a clear relationship. These two statistical measures are then independent. In practice, there may be a lead-lag effect between these two elements. Indeed, some issuers that are beginning to transform their business model to green activities may have positive carbon momentum because of their old system. For instance, increasing green capex has no direct effect on the current carbon footprint, but it will definitively impact the future carbon footprint. Therefore, we expect that this lead-lag effect will be reduced in some years.*

To measure the discrepancy between the benchmark $b(t_0)$ and the optimized portfolio $x^{\star}(t)$, we compute the active share between the weights of these two portfolios. The results are given in Tables 27 and 28 for $\mathcal{C}_0$ and $\mathcal{C}_3(0,10,2)$ constraints. As expected, we observe that the divergence between the benchmark and the decarbonization portfolio increases with the reduction date. In addition, the active share is far more important when implementing a net zero strategy rather than only a decarbonization pathway. On average, we observe a factor of three. Nevertheless, we observe that both approaches lead to relatively high active shares, meaning that decarbonization and portfolio alignment cannot be achieved without significant active costs. If we now compare the net zero portfolio with the corresponding decarbonized portfolio, we notice that the weights are different (see Tables 57 and 58 on page 57). For constraint $\mathcal{C}_0$, the average active share until 2030 is respectively equal to 11% for the case $\mathcal{G}=100\%$ and $\mathcal{CM}^{\star}=-5\%$ and 22% for the case $\mathcal{G}=200\%$ and $\mathcal{CM}^{\star}=-7\%$. These figures become 13% and 35% for constraint $\mathcal{C}_3(0,10,2)$. All these results show that the additional cost of implementing a net zero policy does not only concern the long-term horizon, but they are also important in the short-term horizon. This is a huge difference between the decarbonization dimension and the transition dimension. By construction, this last one implies a spike in the active cost directly at the beginning of the period.

Remark 25. *For the sake of simplicity, we did not impose a constraint on the portfolio turnover and transaction cost, but such optimization problems are specified in Lezmi et al. (2022). Nevertheless, the one-way turnover between dates t and $t+1$ remains low, with an average of 3.2% and 4.5% each year for the decarbonized and net zero portfolios.*

Remark 26. *The previous results are valid for the MSCI World index, a large investment universe. Let us focus on smaller investment universes by considering the MSCI EMU and USA indexes. The*

Figure 24: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB)

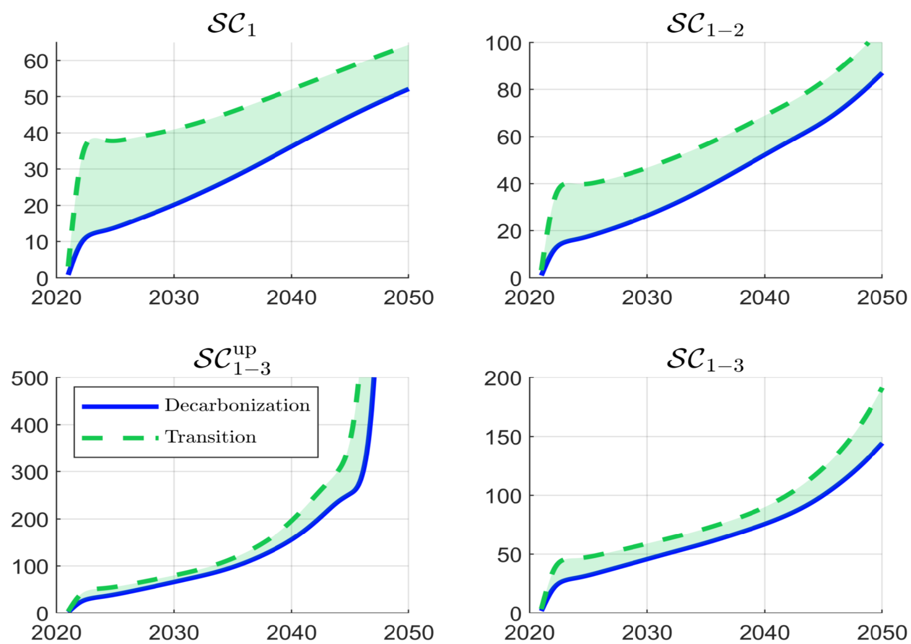


Figure 25: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB)

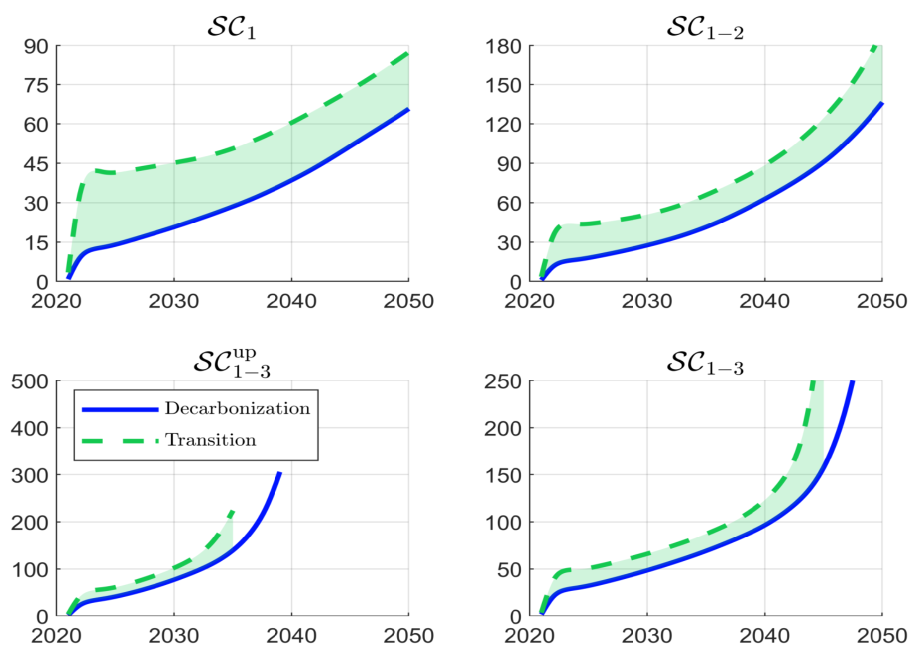


Figure 26: Relationship between the green intensity $\mathcal{GI}_i$ and the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ (MSCI World, Jun. 2022)

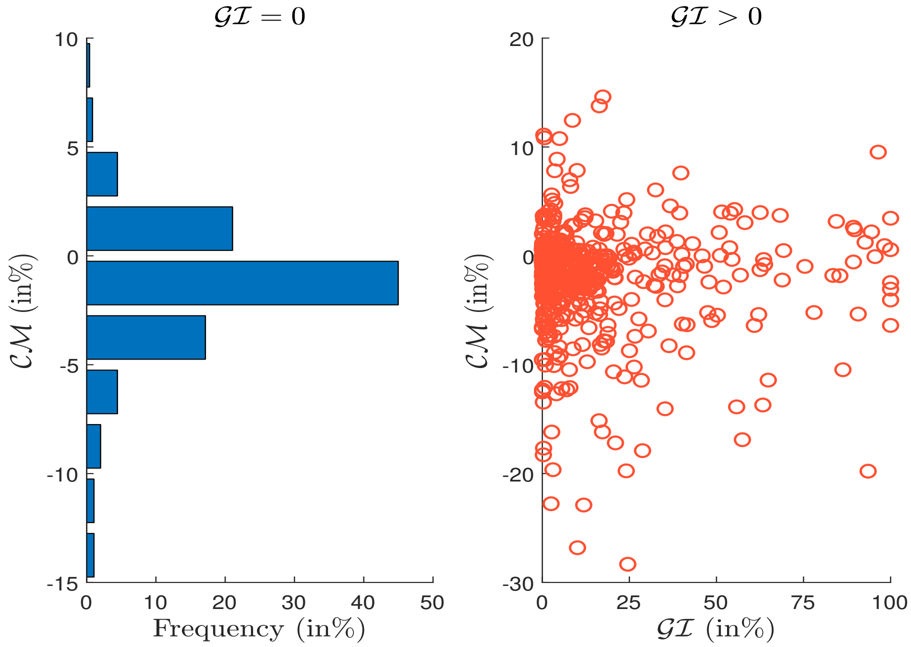


Table 27: Active share (in %) between the benchmark and the optimized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Decarbonized portfolio								
$\mathcal{SC}_1$	3.4	3.7	4.2	4.6	7.2	10.2	13.3	16.0	18.2
$\mathcal{SC}_{1-2}$	4.6	5.2	5.8	6.4	10.4	15.7	21.1	27.8	39.7
$\mathcal{SC}_{1-3}^{\text{up}}$	12.2	14.5	16.8	19.2	30.4	45.1	65.4	82.6	
$\mathcal{SC}_{1-3}$	9.2	10.3	11.3	12.3	18.3	24.1	32.2	45.0	60.7
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	12.1	12.2	12.4	12.6	13.6	15.7	17.7	19.7	21.5
$\mathcal{SC}_{1-2}$	12.4	12.7	12.9	13.3	16.3	20.4	25.2	33.0	44.0
$\mathcal{SC}_{1-3}^{\text{up}}$	16.9	18.9	21.1	23.2	33.9	51.4	71.2	92.7	
$\mathcal{SC}_{1-3}$	14.1	14.8	15.5	16.4	21.4	27.5	37.4	53.0	71.2
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	22.5	22.6	22.7	22.7	23.4	24.7	26.0	27.2	28.5
$\mathcal{SC}_{1-2}$	22.7	22.9	23.0	23.3	25.6	28.5	32.5	40.1	49.8
$\mathcal{SC}_{1-3}^{\text{up}}$	25.2	26.7	28.4	30.2	39.9	58.4	75.4	95.7	
$\mathcal{SC}_{1-3}$	23.2	23.5	23.9	24.4	27.5	33.0	45.1	61.7	78.3

Table 28: Active share (in %) between the benchmark and the optimized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Decarbonized portfolio								
$\mathcal{SC}_1$	3.4	3.7	4.1	4.6	7.2	10.3	14.2	18.9	24.4
$\mathcal{SC}_{1-2}$	4.5	5.1	5.7	6.4	10.6	17.3	27.0	38.8	54.7
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	12.4	14.8	17.2	20.0	36.2	58.7			
$\mathcal{SC}_{1-3}$	9.2	10.2	11.2	12.4	19.8	28.9	40.3	62.2	
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	13.8	14.0	14.2	14.4	15.8	18.0	21.9	26.5	32.6
$\mathcal{SC}_{1-2}$	14.2	14.5	14.9	15.2	18.5	25.9	34.9	47.6	65.9
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	18.7	20.9	23.3	26.3	44.4	71.5			
$\mathcal{SC}_{1-3}$	15.2	15.9	16.9	18.0	25.0	34.1	50.3	83.2	
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	33.6	33.8	34.1	34.4	36.2	38.3	40.9	46.3	55.3
$\mathcal{SC}_{1-2}$	34.4	35.0	35.5	36.1	38.9	44.8	57.8	73.1	86.9
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	37.4	39.1	40.9	43.5	63.4				
$\mathcal{SC}_{1-3}$	32.4	33.0	33.7	34.7	41.6	54.0	76.0		

results are reported in Figures 52–55 on page 128. Tracking errors for smaller universes become greater in fewer years than for the MSCI World index and we also fail to find solutions sooner. We could separate these results by putting scope 1 and scope 2 alignment on one side and scope 3 on the other. Considering scopes 1 and 2, we observe that, for both universes, our aligned portfolio breaks earlier than for the MSCI World. However, even though the MSCI EMU universe is smaller than the USA one, we can find solutions for a longer period. The reason lies in the distribution of green revenue and carbon momentum, which can be more easily conciliated with the intensity reduction constraint for the EMU. Including scope 3 intensities paints another picture. Although the EMU portfolios have lower tracking errors than those from the USA, larger universes tend to give solutions longer. The fact that we are not able to align our EMU portfolio after 2040 in terms of scope 3 carbon intensities therefore highlights the difficulty of portfolio alignment for a relatively small investment universe.

Preventing greenwashing In finance, greenwashing is the action of making people think an investment is not harmful to the environment while this is not really the case. Intentional or not, greenwashing is a reputational risk for financial institutions. Providing full transparency about a financial process helps to reduce this risk. Therefore, a quantitative top-down approach is useful because the different steps of the process are fully described, in particular the objective function and the different constraints. Nevertheless, a top-down approach is not sufficient because some issuers may be selected or overweighted compared to the benchmark, albeit, they do not meet all the conditions of a net zero investment policy. Of course, we can always define an optimization problem by increasing the number of constraints. However, too many of them may produce no solution. This is why we think that a top-down approach must be based on a few intelligible constraints and it must be completed by an ex-post analysis to avoid greenwashing risks.

Various KPIs of a company should be considered when aligning a portfolio to a net zero trajectory. For instance, tracking error minimization can lead to the inclusion of companies that do not actually meet the emission reduction objective. For example, a company with a positive carbon emission momentum can be overweighted compared to the benchmark, as only the carbon intensity momentum is taken into account in the constraints. Similarly, if the optimization is based on scope $\mathcal{SC}_{1-2}$, it can favour companies that better manage this scope than scope $\mathcal{SC}_{1-3}$. Moreover, it seems important to perform a bottom-up analysis of the aligned portfolio to make sure that the selected companies are not subject to climate (or ESG) controversies. The ex-post analysis consists then in analyzing the optimized portfolio and defining a new set of exclusions $x \in \Omega^{\text{ex-post}}_{\mathcal{E}xclusion}(t)$, which generally complete a set of ex-ante or pre-defined exclusions[50] $x \in \Omega^{\text{ex-ante}}_{\mathcal{E}xclusion}$. Therefore, the global optimization problem becomes:

$$
\begin{aligned}
x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \qquad (77)\\
&\text{s.t.} \left\{
\begin{array}{lll}
x \in \Omega_{\mathcal{E}xclusion}(t) = \Omega^{\text{ex-ante}}_{\mathcal{E}xclusion} \cap \Omega^{\text{ex-post}}_{\mathcal{E}xclusion}(t) & \longleftarrow & \text{Exclusion} \\
\mathcal{CI}(t, x) \leq \left(1 - \mathcal{R}(t_0, t)\right) \cdot \mathcal{CI}\left(t_0, b(t_0)\right) & \longleftarrow & \text{Decarbonization} \\
x \in \Omega_{\mathcal{T}ransition}(t) & \longleftarrow & \text{Transition} \\
x \in \Omega_1 \cap \Omega_2(t) & &
\end{array}
\right.
\end{aligned}
$$

6.2.2 Bond portfolios

Dynamic decarbonization We adapt the equity dynamic decarbonization problem to bonds. The solution $x^{\star}(t)$ at time t requires to know the investment universe $b(t)$, the bond risk metrics $\text{DTS}_i(t)$ and $\text{MD}_i(t)$, and the carbon intensity $\mathcal{CI}_i(t)$. In what follows, we perform the exercise assuming that the world does not change[51]. We perform the optimization by considering only the decarbonization pathways of CTB and PAB labels. The results are given in Figures 27 and 28. The DTS tracking risk is not significant and is lower than 6 bps until 2030. This is not the case of the active share risk, since it can reach 20% for the PAB decarbonization pathway in 2030. We also notice that the active share risk is an increasing function of the year and the scope until 2040. After this year, scope $\mathcal{SC}^{\text{up}}_{1-3}$ takes the lead on scope $\mathcal{SC}_{1-3}$. Nevertheless, we do not have the significant gap observed in the case of equities between upstream scope 3 and the other scopes.

Controlling the greenness We apply the transition constraint for different values of $\mathcal{G}$: 0%, 100% and 200%. In Table 29, we present PAB results, but CTB results are comparable and available in the appendix (Table 59 on page 112). We do not report the DTS tracking risk since it is negligible (less than 1 bp for $\mathcal{G} = 100\%$). The active share cost is low and close to zero when the goal is to maintain the greenness of the benchmark. The reason is that most decarbonized portfolios already have a green intensity greater than or equal to that of the benchmark (see Table 21 on page 68). When $\mathcal{G} = 100\%$, the additional cost is between 0.2% and 0.9% until 2030. This cost becomes high when we want to triple the green intensity, and can reach 4.2%.

Integrating the carbon momentum constraint In Table 30, we report some statistics about the carbon momentum. Obviously, the higher the upper bound $\mathcal{CM}^{+}$, the lower the number of excluded issuers. Removing all issuers with positive carbon momentum represents 542 out of 2362 issuers and 23.5% of the benchmark, while only 51 issuers (and 1.5% of the benchmark) are discarded when $\mathcal{CM}^{+}$ is equal to 5%.

We suppose that $\mathcal{G}$ is equal to 100%. Since we have seen that the additional tracking cost (DTS and AS) is small when we control the green intensity, we add the momentum exclusion constraint to the previous optimization problem. Table 31 shows the additional active share cost after the greenness control. When $\mathcal{CM}^{+}$ is equal to 0%, this cost until 2030 is above 20% for the scope $\mathcal{SC}_1$ and 15% for the scope $\mathcal{SC}_{1-3}$. As expected, excluding issuers with positive carbon momentum has

[50]Generally, asset managers exclude worst-in-class ESG issuers, companies with a large business on thermal coal and oil, etc.

[51]This implies that $\mathcal{CI}_i(t) = \mathcal{CI}_i(t_0)$, $b(t) = b(t_0)$, $\text{DTS}_i(t) = \text{DTS}_i(t_0)$ and $\text{MD}_i(t) = \text{MD}_i(t_0)$.

Figure 27: Duration-times-spread of dynamic decarbonized portfolios in bps (Global Corp., Jun. 2022)

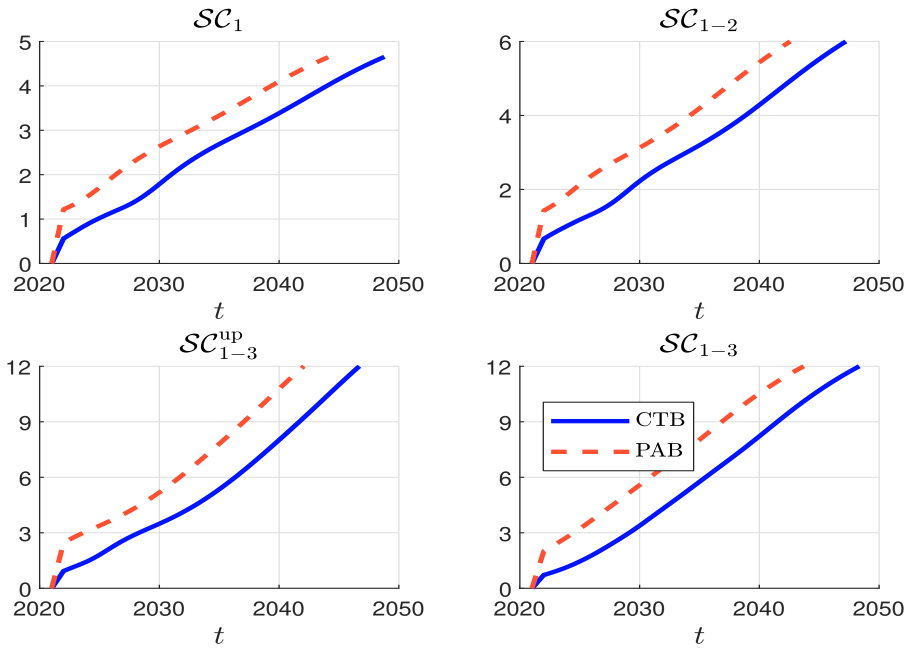


Figure 28: Active share of dynamic decarbonized portfolios in % (Global Corp., Jun. 2022)

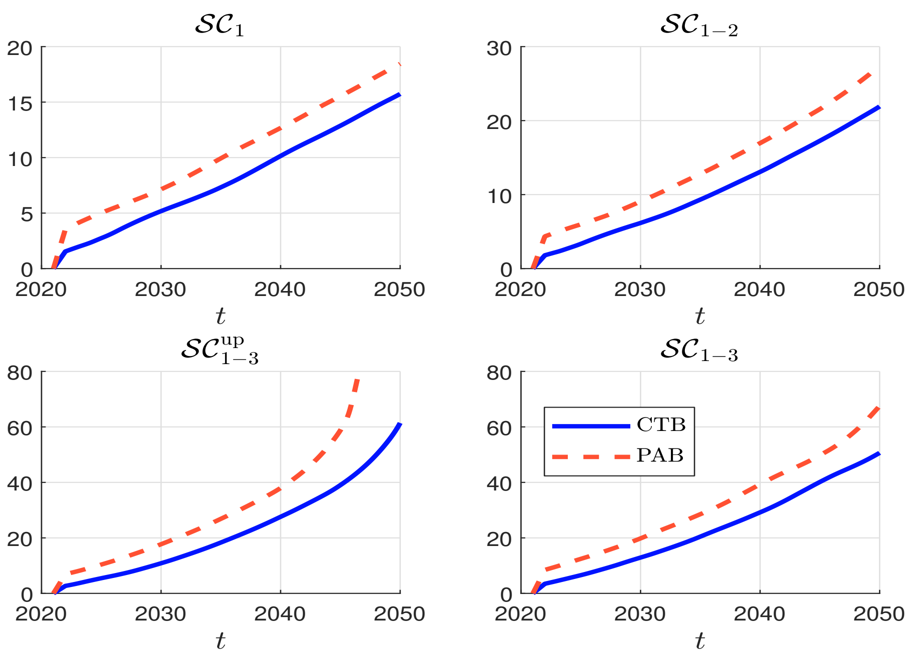


Table 29: Additional active share cost in % when we control the green intensity (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{G} = 0\%$				
$\mathcal{SC}_1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathcal{SC}_{1-2}$	0.1	0.1	0.1	0.1	0.2	0.1	-0.2	-0.3	-0.2
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.1	0.1	0.2	0.2	0.0	-0.2	-0.1	0.2	0.3
$\mathcal{SC}_{1-3}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
					$\mathcal{G} = 100\%$				
$\mathcal{SC}_1$	0.9	0.6	0.4	0.3	0.2	0.2	0.0	-0.2	-0.4
$\mathcal{SC}_{1-2}$	0.6	0.4	0.4	0.4	0.4	0.2	0.0	-0.1	0.1
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.3	0.4	0.4	0.5	0.5	0.5	0.4	1.8	0.3
$\mathcal{SC}_{1-3}$	0.4	0.3	0.3	0.3	0.4	0.5	0.3	-0.6	-3.6
					$\mathcal{G} = 200\%$				
$\mathcal{SC}_1$	4.2	3.8	3.5	3.2	2.0	0.8	0.2	0.0	-0.3
$\mathcal{SC}_{1-2}$	3.7	3.3	3.0	2.7	1.5	1.0	0.4	0.3	0.6
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	2.3	1.8	1.4	1.3	1.3	1.6	1.6	3.8	0.3
$\mathcal{SC}_{1-3}$	1.6	1.2	1.0	0.8	0.7	0.9	0.7	0.0	-2.2

Table 30: Statistics of the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ (Global Corp., Jun. 2022)

Statistic	Median	Negative	Positive	$\mathcal{CM}_i <$		$\mathcal{CM}_i >$	
				-10%	-5%	$+5\%$	$+10\%$
Frequency (in %)	-1.3	77.1	22.9	3.3	14.9	2.2	0.8
Weight (in %)		76.5	23.5	4.2	13.4	1.5	0.9

a substantial cost compared to the cost of doubling the green intensity. When we set $\mathcal{CM}^+ = 5\%$, the cost is negligible since these issuers represent about 1.5% of the benchmark.

Table 31: Additional active share cost in % when we implement a momentum exclusion approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^+ = 0\%$								
$\mathcal{SC}_1$	20.5	20.6	20.5	20.5	20.1	18.6	17.5	16.7	16.1
$\mathcal{SC}_{1-2}$	20.1	20.0	19.8	19.7	18.5	16.5	15.1	13.6	11.4
$\mathcal{SC}_{1-3}^{\text{up}}$	18.6	18.1	17.6	17.0	13.8	11.6	9.7	4.1	0.3
$\mathcal{SC}_{1-3}$	17.2	16.8	16.5	16.3	14.6	13.2	12.5	12.3	13.9
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^+ = 5\%$								
$\mathcal{SC}_1$	0.6	0.7	0.9	1.0	1.0	0.9	0.9	0.8	0.7
$\mathcal{SC}_{1-2}$	0.7	0.8	0.9	0.9	0.6	0.6	0.5	0.3	0.1
$\mathcal{SC}_{1-3}^{\text{up}}$	0.9	0.8	0.7	0.7	0.5	0.3	0.2	0.1	0.0
$\mathcal{SC}_{1-3}$	1.1	1.1	1.1	1.0	0.8	0.6	0.5	0.3	0.0

Remark 27. *In Table 61 on page 113, we report additional DTS cost because it is the only case where it is significant. Indeed, when $\mathcal{G} = 100\%$ and $\mathcal{CM}^+ = 0\%$, we can observe an additional DTS cost that is close to 5 bps.*

In Table 63 on page 114, we have reported the carbon momentum difference $\Delta\mathcal{CM}(t) = \mathcal{CM}\left(t, x^\star(t)\right) - \mathcal{CM}\left(t, b(t)\right)$. We note that the difference is negative with a null $\mathcal{CM}^+$ and frequently decreases with the years. When we set $\mathcal{CM}^+$ to 5%, the story is different. The decarbonized portfolio often shows a worse carbon reduction trend than the benchmark reference, and the difference may even increase with the years. To ensure a better trajectory for the decarbonized portfolio, we change the momentum approach and use the global momentum constraint $\mathcal{CM}^{\mathcal{L}ong}(t, x) \leq \mathcal{CM}^\star$. This second strategy is less harmful in active share, especially compared to $\mathcal{CM}^+ = 0\%$. Indeed, if we apply $\mathcal{CM}^\star = -5\%$ and $\mathcal{CM}^\star = -7\%$, the difference in active share remains below 1.4% (Table 32).

Table 32: Additional Active share cost in % of a global momentum threshold approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	0.6	0.7	0.7	0.7	0.7	0.2	0.2	0.2	0.2
$\mathcal{SC}_{1-2}$	0.6	0.6	0.6	0.6	0.3	0.1	0.1	0.2	0.1
$\mathcal{SC}_{1-3}^{\text{up}}$	0.5	0.5	0.5	0.4	0.2	0.2	0.1	0.4	−0.2
$\mathcal{SC}_{1-3}$	0.3	0.3	0.4	0.4	0.4	0.4	0.7	0.9	1.9
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	1.4	1.4	1.3	1.3	1.4	0.8	0.4	0.4	0.4
$\mathcal{SC}_{1-2}$	1.2	1.1	1.1	1.1	1.0	0.4	0.4	0.4	0.3
$\mathcal{SC}_{1-3}^{\text{up}}$	0.9	0.9	0.9	0.8	0.5	0.5	0.4	0.0	−0.1
$\mathcal{SC}_{1-3}$	0.6	0.7	0.7	0.7	0.7	0.8	1.2	1.7	3.5

Remark 28. *Results for the CTB pathway are shown in Tables 60, 62 and 64 on page 113.*

Preventing greenwashing We have already presented the ex-post exclusion approach on page 78. In this paragraph, we explore other approaches. For instance, we can impose that the weight in the aligned portfolio can not exceed the weight in the benchmark for issuers with a positive carbon momentum. For each issuer j, we note $\mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CE}, \mathcal{SC})$ and $\mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC})$ the carbon emission and intensity momentum measures for the corresponding scope $\mathcal{SC}$. Let $\mathcal{NCM}_j$ be the total number of positive carbon momentum:

$$\mathcal{NCM}_j = \sum_{\mathcal{SC}=\mathcal{SC}_1, \mathcal{SC}_{1-2}, \mathcal{SC}_{1-3}} \left\{ \mathbb{1}\left\{ \mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CE}, \mathcal{SC}) \right\} + \mathbb{1}\left\{ \mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC}) \right\} \right\} \tag{78}$$

$\mathcal{NCM}_j$ takes its values between 0 and 6. We also define $\mathcal{NCM}'_j$ when we only consider the carbon intensity momentum:

$$\mathcal{NCM}'_j = \mathbb{1}\left\{ \mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC}_1) \right\} + \mathbb{1}\left\{ \mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC}_{1-2}) \right\} + \mathbb{1}\left\{ \mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC}_{1-3}) \right\} \tag{79}$$

In this case, $\mathcal{NCM}'_j$ takes its values between 0 and 3.

Table 33: Frequency and weight of positive carbon momentum (Global Corp., Jun. 2022)

$\mathcal{NCM}_j$	$\mathcal{NCM}'_j$ Frequency in (%)				$\mathcal{NCM}'_j$ Weight in (%)			
	0	1	2	3	0	1	2	3
0	32.26				32.84			
1	10.75	0.76			8.67	1.53		
2	12.49	1.14	0.59		8.93	3.64	0.85	
3	14.27	1.61	0.47	2.54	12.51	4.53	0.66	3.38
4		2.84	2.20	0.47		3.01	3.01	0.56
5			4.57	0.47			5.32	1.04
6				12.57				9.51
Total	69.77	6.35	7.83	16.05	62.94	12.72	9.85	14.49

In Table 33, we report the frequencies of $\left(\mathcal{NCM}_j, \mathcal{NCM}'_j\right)$. We notice that less than one-third of issuers have six negative carbon trends, implying that the matrix of carbon trends is exactly this one:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\searrow$	$\searrow$	$\searrow$
$\mathcal{CI}$	$\searrow$	$\searrow$	$\searrow$

12.57% of issuers have six positive carbon trends:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\nearrow$	$\nearrow$	$\nearrow$
$\mathcal{CI}$	$\nearrow$	$\nearrow$	$\nearrow$

This implies that about 55% of issuers have both positive and negative trends. Among them, 14.27% of issuers are in the following configuration:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\nearrow$	$\nearrow$	$\nearrow$
$\mathcal{CI}$	$\searrow$	$\searrow$	$\searrow$

whereas 2.54% of issuers are in the opposite configuration:

	$\boldsymbol{\mathcal{SC}}_1$	$\boldsymbol{\mathcal{SC}}_{1-2}$	$\boldsymbol{\mathcal{SC}}_{1-3}$
$\boldsymbol{\mathcal{CE}}$	$\searrow$	$\searrow$	$\searrow$
$\boldsymbol{\mathcal{CI}}$	$\nearrow$	$\nearrow$	$\nearrow$

Finally, about 28% of issuers are in the other configurations.

Let us assume that we use the carbon intensity trend based on the scope $\boldsymbol{\mathcal{SC}}_{1-3}$ to define the self-decarbonization constraint in the optimization problem, the bad case is the following configuration:

	$\boldsymbol{\mathcal{SC}}_1$	$\boldsymbol{\mathcal{SC}}_{1-2}$	$\boldsymbol{\mathcal{SC}}_{1-3}$
$\boldsymbol{\mathcal{CE}}$	$\nearrow$	$\nearrow$	$\nearrow$
$\boldsymbol{\mathcal{CI}}$	$\nearrow$	$\nearrow$	$\searrow$

For these issuers, we want to underweight their allocation relative to the benchmark. More generally, we can define the following constraint[52]:

$$\Omega_{\boldsymbol{\mathcal{G}}reen\boldsymbol{\mathcal{W}}ash} = \left\{\boldsymbol{\mathcal{NCM}}_j > 0 \implies \sum\nolimits_{i \in \boldsymbol{\mathcal{I}}ssuer(j)} x_i \leq \sum\nolimits_{i \in \boldsymbol{\mathcal{I}}ssuer(j)} b_i\right\} \tag{81}$$

Table 34 shows the impact of $\Omega_{\boldsymbol{\mathcal{G}}reen\boldsymbol{\mathcal{W}}ash}$ on active share cost. Just as it is below 0.7% for $\boldsymbol{\mathcal{SC}}_1$, it remains below 1% until 2030 for the other scopes. Applying the greenwashing constraint $\Omega_{\boldsymbol{\mathcal{G}}reen\boldsymbol{\mathcal{W}}ash}$ on $\boldsymbol{\mathcal{NCM}}'_j$ yields lower additional costs due to a lower frequency of constrained issuers. For instance, these costs would remain below 0.4% for the three scopes.

Table 34: Additional Active share cost in % of the constraint $\Omega_{\boldsymbol{\mathcal{G}}reen\boldsymbol{\mathcal{W}}ash}$ (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\boldsymbol{\mathcal{G}} = 100\%$ and $\boldsymbol{\mathcal{CM}}^\star = -5\%$								
$\boldsymbol{\mathcal{SC}}_1$	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.7
$\boldsymbol{\mathcal{SC}}_{1-2}$	0.2	0.2	0.2	0.2	0.2	0.5	1.0	2.5	6.4
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	0.2	0.2	0.2	0.3	0.9	2.2	8.6	2.2	−2.1
$\boldsymbol{\mathcal{SC}}_{1-3}$	0.4	0.5	0.5	0.5	1.0	1.7	3.1	6.6	20.4
	$\boldsymbol{\mathcal{G}} = 100\%$ and $\boldsymbol{\mathcal{CM}}^\star = -7\%$								
$\boldsymbol{\mathcal{SC}}_1$	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.3	0.5
$\boldsymbol{\mathcal{SC}}_{1-2}$	0.2	0.2	0.2	0.1	0.1	0.4	1.0	2.3	6.3
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	0.2	0.2	0.2	0.2	0.9	2.2	8.0	4.2	0.1
$\boldsymbol{\mathcal{SC}}_{1-3}$	0.5	0.5	0.5	0.5	1.0	1.7	3.0	6.5	20.5

6.2.3 Diversification and liquidity risk

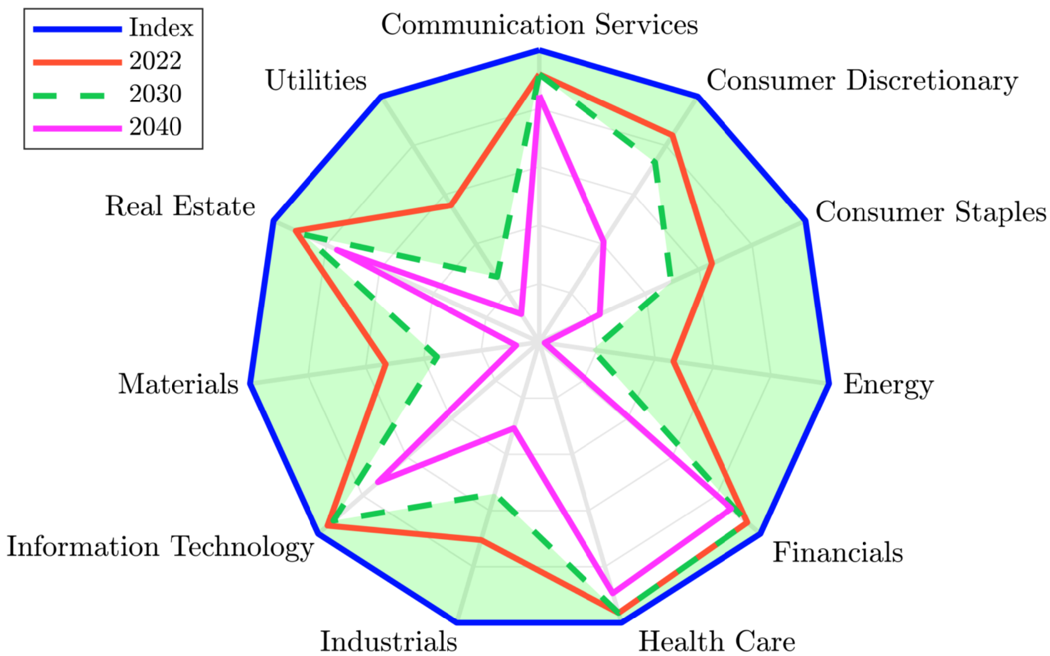
In practice, a lot of constraints can be used in the construction of aligned portfolios. We have previously seen that the tracking error cost can be significant and that the solution may also not exist for long time horizons. Since some assets are excluded from the net zero portfolio, this one

[52] An alternative approach is to constraint each bond of these issuers:

$$\Omega_{\boldsymbol{\mathcal{G}}reen\boldsymbol{\mathcal{W}}ash} = \left\{\boldsymbol{\mathcal{NCM}}_j > 0 \implies \forall i \in \boldsymbol{\mathcal{I}}ssuer\,(j) : x_i \leq b_i\right\} \tag{80}$$

may be more concentrated than the benchmark. Therefore, we might face not only a diversification risk, but also a liquidity risk. These risks will be reduced if the economy decarbonizes itself in the coming years. Nevertheless, we are not immune that carbon emissions keep increasing in the short term. In this case, the solutions will be very sensitive to the gap between the carbon objective of net zero portfolios and the carbon footprint of the economy.

Figure 29: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



To illustrate the shrinkage risk of the investment universe, we compute the number of selected stocks per sector for each optimized portfolio and divide these figures by the corresponding stocks number in the index[53]. In the case of the scope $\boldsymbol{\mathcal{SC}}_{1-3}$, the radar charts of these frequencies are reported[54] in Figures 29 and 30. We observe that the investment universe is shrunk at the first date. The green area represents the removed part by 2030. With the exception of the Communication Services, Financials, Health Care, Information Technology and Real Estate sectors, the investment in the other sectors is concentrated on few stocks. This shrinkage effect can also be observed for small investment universes[55].

By construction, the shrinkage of the investment universe worsens if we add other constraints. For instance, the impact of the momentum exclusion constraint is illustrated in Figure 31. In this case, we complete the set of constraints by the threshold constraint $\left\{\boldsymbol{\mathcal{CM}}_i^{\mathcal{L}ong}(t) \geq 0 \Rightarrow x_i = 0\right\}$, meaning that we exclude issuers with a positive carbon trend. We notice that the investment universe is highly reduced even from the first year. This type of high impact is also observed when we compare Case #1: $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$ and Case #2: $\boldsymbol{\mathcal{G}} = 200\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -7\%$ (see Figures 68 and 69 on page 137).

[53] For instance, if the frequency is equal to 25% for the Energy sector, this means that the optimized portfolio has selected 25% of Energy stocks and removed 75% of the Energy investment universe.

[54] The results for the different scopes are shown on pages 131–135.

[55] See Figures 64 and 65 on page 135 for the MSCI EMU index.

Figure 30: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

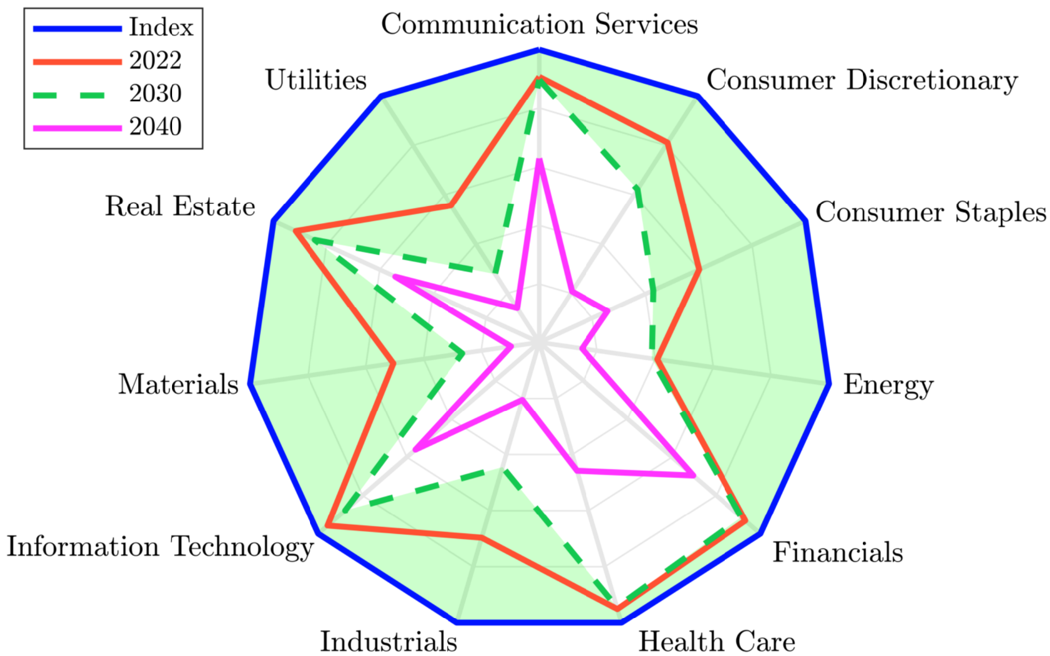


Figure 31: Impact of momentum exclusion on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

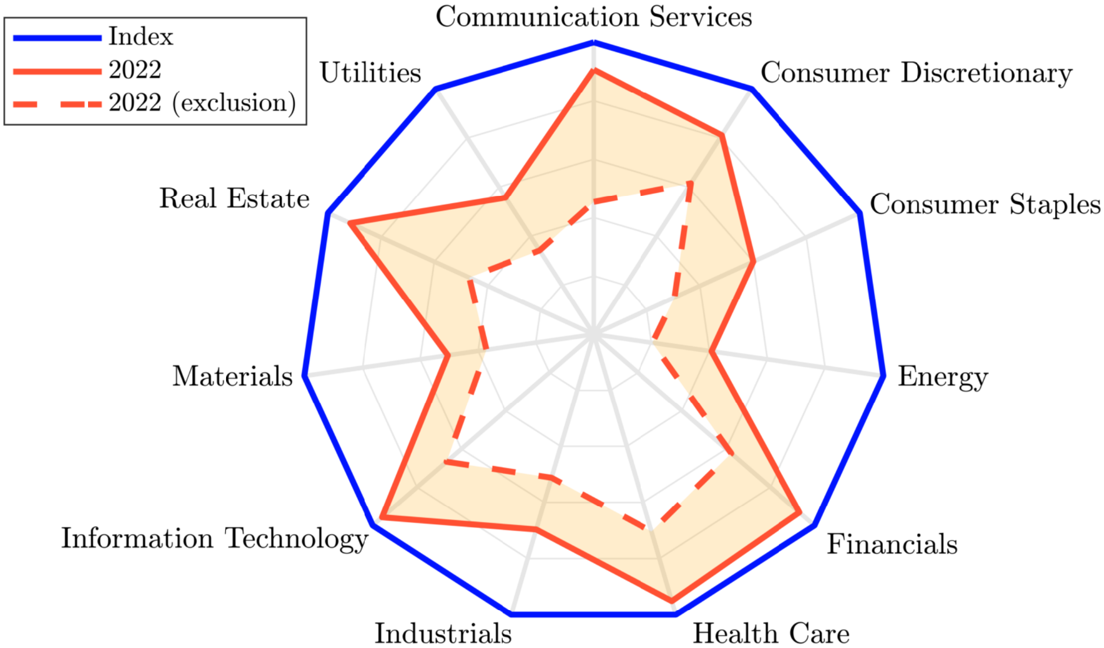


Figure 32: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

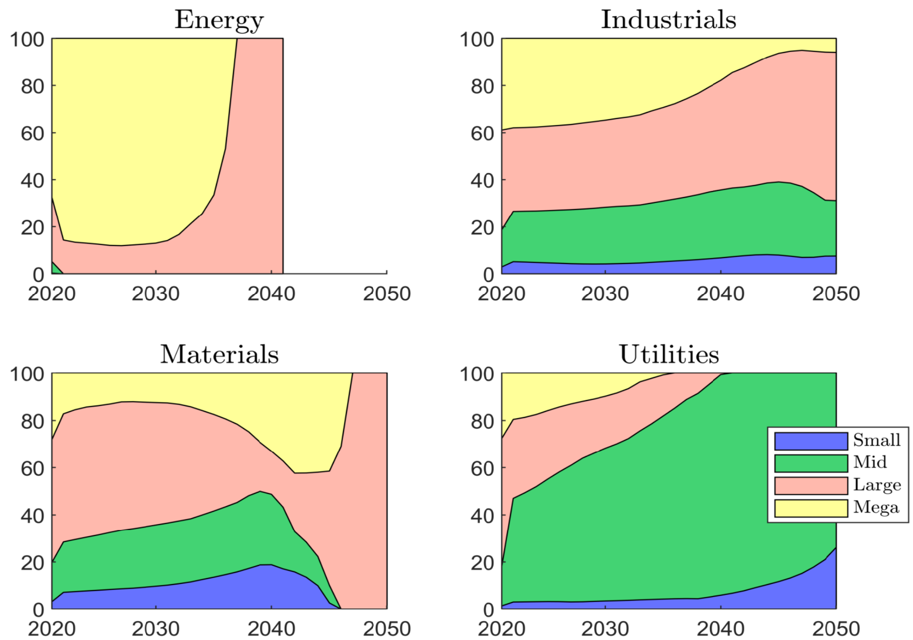
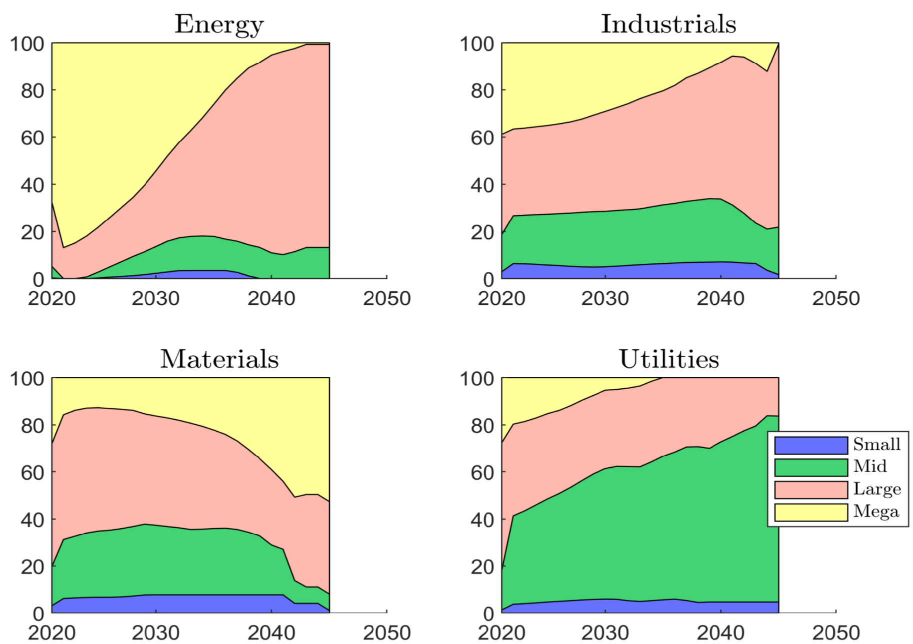


Figure 33: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



Remark 29. *These results show that we cannot reduce the cost of net zero investing to the cost of tracking risk. As seen above, there is also a cost of diversification risk. In this dissertation, we do not consider the cost of liquidity risk, but it does not mean that it is negligible. To give an idea, we have calculated the breakdown of the allocation with respect to the market capitalization. We consider four buckets: small-cap (below* \$4.5 *bn, mid-cap (between* \$4.5 *and* \$12.5 *bn), large-cap (between* \$12.5 *and* \$50 *bn and big-cap (above* \$50 *bn). The results for the four strategic sectors (Energy, Industrials, Materials and Utilities) are reported in Figures 32 and 33 (See Figures 70–73 on page 138 for the other sectors). We notice that the allocation to large- and mid-cap buckets is reduced while the allocation to small- and micro-cap buckets increases over time.*

Regarding bond portfolios We measure the issuer concentration by the inverse of the Herfindahl index. This indicator defines the number of bets, or the degrees of freedom of portfolio weights. It is equal to one if the portfolio is concentrated on one asset. Conversely, it is equal to the number of assets for an equally-weighted portfolio, which is the least concentrated portfolio in terms of weights. The current benchmark is comprised of 2 362 companies corresponding to 342 equally-weighted issuers. The benchmark is far from being highly diversified as the first quintile of issuers represents 77.2% of the benchmark weights while the last quintile corresponds to 1.2%. Table 35 displays the number of bets of optimized portfolios. This number decreases with the year, indicating more and more concentrated portfolios. This is especially true for scope $\boldsymbol{\mathcal{SC}}_{1-3}$, where the number of bets is divided by a factor of 2.5 by 2030 and 5 by 2035. The evolution of the top 10 issuers' weights gives another picture of the extent of the diversification (Table 36). On average, we observe that the concentration in the top 10 issuers is multiplied by a factor of 2 in 2030 and 5 in 2045 if we focus on scope $\boldsymbol{\mathcal{SC}}_{1-3}$.

Table 35: Number of bets (Global Corp., Jun. 2022, PAB)

Scope	Index	2022	2023	2024	2025	2030	2035	2040	2045	2050
		$\boldsymbol{\mathcal{G}} = 100\%$ and $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$								
$\boldsymbol{\mathcal{SC}}_1$	342	265	265	255	256	249	213	200	161	130
$\boldsymbol{\mathcal{SC}}_{1-2}$		259	257	255	254	205	187	156	122	66
$\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$		246	229	202	191	121	75	34	19	8
$\boldsymbol{\mathcal{SC}}_{1-3}$		227	216	208	198	131	69	34	22	9
		$\boldsymbol{\mathcal{G}} = 100\%$ and $\boldsymbol{\mathcal{CM}}^{\star} = -7\%$								
$\boldsymbol{\mathcal{SC}}_1$	342	225	228	228	222	226	180	167	148	121
$\boldsymbol{\mathcal{SC}}_{1-2}$		229	222	217	222	205	169	133	110	71
$\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$		229	216	194	181	118	74	47	23	6
$\boldsymbol{\mathcal{SC}}_{1-3}$		197	190	191	180	124	65	43	24	11

Table 36: Top 10 issuers' weight in % (Global Corp., Jun. 2022, PAB)

Scope	Index	2022	2023	2024	2025	2030	2035	2040	2045	2050
		$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$		13.4	13.4	13.7	13.5	13.6	15.3	16.4	19.5	21.9
$\mathcal{SC}_{1-2}$	10.9	13.6	13.6	13.7	13.6	15.8	17.5	19.5	22.6	30.8
$\mathcal{SC}_{1-3}^{\mathrm{up}}$		13.8	14.2	15.8	16.3	21.9	29.6	42.2	56.6	88.3
$\mathcal{SC}_{1-3}$		15.3	15.6	16.4	16.8	21.7	29.9	42.2	55.0	80.8
		$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$		14.9	14.7	14.2	14.5	14.8	17.0	18.0	19.7	22.4
$\mathcal{SC}_{1-2}$	10.9	14.5	14.6	14.9	15.0	16.0	18.1	20.9	24.3	31.1
$\mathcal{SC}_{1-3}^{\mathrm{up}}$		14.4	15.4	16.3	16.7	23.1	29.7	40.4	57.2	90.9
$\mathcal{SC}_{1-3}$		16.8	17.4	17.2	18.1	22.2	30.8	41.4	53.6	79.4

6.3 Core-satellite approach

Another solution to build a net zero portfolio is to implement a core-satellite approach. Indeed, a net zero investment strategy implies two building blocks. The first building block concerns the decarbonization of the portfolio while the objective of the second building block is to finance the transition to a low-carbon economy. In this context, the decarbonization portfolio plays the role of a core investment, whereas the transition portfolio is like a thematic portfolio or a satellite basket. Typically, the underlying idea of a core-satellite strategy is to boost a passive portfolio with actively managed strategies or 'exotic' asset classes that have the potential to enhance risk-adjusted returns. In our case, the purpose of the core-satellite strategy is to boost the greenness or the alignment of a decarbonized portfolio with respect to net zero objectives.

Table 37: The core-satellite approach

Decarbonized portfolio		**Transition portfolio**
• Low-carbon portfolio		• Climate/green solutions
• Decarbonization pathway(**s**)		• Financing the transition
• Top-down approach	+	• Bottom-up approach
• Portfolio optimization		• Security selection
• Carbon measures		• $\neq$ Carbon measures
• Net zero **carbon** metrics		• Net zero **transition** metrics
$\mathbf{1-\alpha(t)}$		$\boldsymbol{\alpha}\mathbf{(t)}$

The portfolio construction is defined in Table 37. As we have already seen, decarbonization is typically a top-down approach, whereas transition is more a bottom-up approach, or a security selection process. The core-satellite approach circumvents the problem of the negative correlation between decarbonization and transition in the short-term. It also reduces the complexity of dealing with many constraints and many climate risk measures that are not always compatible. Moreover, portfolio managers have extensive experience in portfolio decarbonization and its associated metrics. They don't need to have a strong background about climate investing. Therefore, portfolio decarbonization can be implemented on a massive scale. This is not the case with the transition basket, which requires specialized portfolio managers. These last ones must understand net zero challenges, metrics and concepts such as self-decarbonization, green capex or climate taxonomy. In this case, it is obvious that traditional carbon metrics are not adapted to the transition dimension. For instance, if we consider investment in hydrogen solutions, it may have a high carbon footprint. This is not incompatible with the transition dimension if this investment is helpful in building a low-carbon economy in the future. Therefore, the reporting of the transition basket must be based more on impact investing and net zero transition measures than on traditional carbon footprints.

We may wonder why the transition portfolio corresponds to the satellite portfolio. Mainly because we have seen that transition and green activities are today a small portion of the investment universe. From a strategic asset allocation viewpoint, allocating 10% of a global portfolio to green solutions is already a big progress. But it is important to notice that the proportion $\alpha(t)$ allocated to the transition dimension is time-varying and must increase with the enlargement of the green investment universe in the future.

7 Conclusion

In this dissertation dedicated to net zero investing, we break down a net zero investment policy into two dimensions: decarbonization and transition. First, we present the two families of metrics needed to implement such a policy. While we assess the first dimension through traditional carbon footprint measures and a decarbonization pathway, we suggest some metrics to evaluate the ability to finance the transition to a low-carbon economy and the willingness of issuers to participate in the net zero journey. In particular, the green revenue share is an interesting proxy for assessing this second dimension because it grants high data coverage. Since portfolio alignment is a dynamic process, we also highlight the need to consider static and forward-looking metrics for decarbonization and transition. To this extent, we use carbon trends for portfolio decarbonization and emphasize the lack of forward-impactful transition data. For example, green capex — in addition to being seldom disclosed — does not always lead to patent filing and even less to commercialization. Beyond these metrics, we introduced key concepts to better understand net zero investment portfolios. These concepts mainly encompass the $\mathcal{PAC}$ framework, and in particular the participation pillar. Indeed, net zero investing implies a dynamic view of portfolio decarbonization. Therefore, we propose using carbon momentum measures to gauge the self-decarbonization ability of issuers. A portfolio could only be labeled net zero if it reaches some minimum requirements of self-decarbonization. Indeed, if the decarbonization pathway is achieved only because the fund manager rebalances the portfolio at a given frequency to obtain a higher reduction rate, the decarbonization pathway followed by the portfolio is purely exogenous and is explained by the rebalancing process. In the case of a net zero portfolio, a part of the decarbonization pathway must become endogenous and explained by self-decarbonization. In this approach, decarbonization is not due to external factors (e.g., the rebalancing scheme), but internal factors also participate. This is one of the two main differences between a net zero investment policy and a low-carbon strategy, the former being to focus on the transition pillar, as seen previously.

Alongside our suggestions, we implement an optimization-based approach for aligning a portfolio by integrating various constraints based on the previous metrics. Generally, we use three constraints. The first one targets the time-varying decarbonization rate, the second imposes a minimum green revenue share, and the last one uses carbon momentum metrics to forecast the self-decarbonization rate. If we consider the classical framework that consists in replicating a benchmark and controlling the tracking risk, our empirical results show the following lessons.

The first lesson concerns the sensitivity of the solution to parameters and data. In particular, fund managers need to be careful when they select the carbon scope metric to assess the decarbonization rate. Net zero only makes sense if it concerns a closed system. Therefore, scope 3 emissions must be considered to align a portfolio with respect to a net zero scenario. The issue is that we observe a lack of data reliability on scope 3 emissions today. Nevertheless, it is important that asset owners and managers begin to use scope 3 in order to create incentives to improve data quality. These incentives concern several actors: regulators, issuers, and data providers. However, including scope 3 increases the tracking error risk, particularly with the upstream emissions. Similarly, the solution is highly dependent on the figure we target for the green revenue share and the carbon momentum rate we would like to achieve for the self-decarbonization level. Fund managers must then be careful because too much ambition in the short term implies that there may be no solution in the medium term to the optimization problem. The no-solution issue depends on the relative speed of the portfolio's decarbonization pathway with respect to the economy's decarbonization pathway and the initial starting point.

The second main result is that portfolio decarbonization and portfolio alignment give different solutions. In particular, decarbonizing a portfolio is easier than aligning a portfolio. We show that decarbonizing along CTB or PAB pathways never leads to exploding tracking errors until 2030. In fact, the real issue of the decarbonization exercise lies in the diversification and liquidity risk an investor might face. These results are amplified when we add the transition dimension into the optimization program. Along with a higher tracking error cost, there is no guarantee that a solution always exists. Besides, introducing the transition pillar emphasizes the difficulty of choosing a proper set of constraints for net zero portfolios, because some metrics can be negatively correlated with others. Portfolio decarbonization is systematically a strategy that is long on Financial issuers and short on Energy, Materials and Utilities issuers. Therefore, we have a situation where the transition dimension of a decarbonized portfolio is weaker than that of the benchmark portfolio as green solutions are also located in carbon-intensive sectors. Thus, it is crucial to distinguish between issuers with a high carbon footprint that will not participate in the transition and those that will reduce their carbon emissions and find low-carbon solutions. Since the transition dimension is multi-faceted, professionals are tempted to multiply the transition metrics. This is not always a good idea because these metrics may be independent in the short run. For example, we observe no current relationship between carbon momentum and green revenue share. However, we can assume that these two metrics will be correlated in the long term when the economy will be on the right track to reach net zero. Since many independent metrics do not ensure the existence of a solution, it is better to concentrate on a small number of transition constraints and to understand the objective of each one. True to the saying that "*less is more*", a concise problem for defining net zero is more useful than a complex patchwork and a diffuse stack of criteria. In this last case, the balance is always difficult to find.

The third main result is that portfolio decarbonization and alignment are two exclusion processes. This means it is quite impossible to achieve net zero alignment without allowing the algorithm to exclude companies from the benchmark. For instance, the optimization program does not generally find a solution when imposing lower bounds other than zero. Therefore, some key actors of the transition such as Energy and Utilities companies unfortunately disappear. Moreover, imposing sector neutrality may lead to similar problems finding a solution. The exclusion process that we observe at both issuer and industry levels raises the question of benchmarking. Indeed, if portfolio decarbonization can be viewed as a tilt of the benchmark portfolio, portfolio alignment may imply a significant shrinkage of the investment universe. As such, defining the net zero investing benchmark is complex because it is too far from business-as-usual investing. Of course, in the long run, we will observe a convergence between net zero and market portfolios when the world economy reaches net zero emissions. But, in the short term, the gap remains wide, and an alternative benchmark choice is an important issue for all net zero investors.

Another lesson concerns the question of greenwashing, which is a key challenge of net zero investment. Here, we are referring to explicit and deliberate greenwashing, which is a mis-selling risk from a regulatory viewpoint, but rather unintentional greenwashing, which is more of a misinterpretation risk. This risk occurs when (1) the practices and definitions are not unique and (2) the practices and definitions change over time. Regulators have not yet defined a normative and comprehensive framework for net zero investment policies. As a result, two investors may have two different visions about net zero, implying that they do not use the same criteria. Moreover, as we said previously, it is really difficult to manage all aspects of a top-down optimization process. Therefore, it is always easy to analyze the net zero portfolio of an investor and to find some issuers that are not net zero using other criteria. For example, our optimization model uses intensity-based carbon momentum including scope 3, because the decarbonization pathway is expressed with the carbon intensity measure and scope 3. We could also use emission-based carbon momentum or another scope. We can multiply the criteria but as we explained before, the no-solution risk increases. Moreover, another dimension that is difficult to integrate in a top-down approach is the engagement and ESG stewardship of asset owners and managers. Therefore, we need a bottom-up analysis of the issuers that make up the net zero portfolio. The fund manager must validate each constituent. In a sense, building a net zero portfolio is an active management strategy, and the fund manager must be convinced that each exposure is justified. Applying a bottom-up check will then reduce the risk of greenwashing controversies.

Contrary to some academic publications, we find that the tracking error cost may be significant even in the short term. This is particularly true for equity portfolios and small investment universes, but much less for bond portfolios. At first sight, this result may be surprising because there is no reason that net zero impacts equity and fixed-income markets differently. In fact, there are two explanations. First, the structure of equity and bond indices are different, with a more balanced allocation across sectors and a high exposure to Financial issuers for the latter. Second, bond indices are highly affected by new fresh capital, whereas equity indices are sticky to the stock of existing capital (or old capital). This is because the primary bond market is very active, implying a significant impact on the secondary market. This is not the case in the equity market, where IPOs and capital increases only represent a small fraction of the secondary market. This implies that portfolio holdings change faster for bond indices than equity indices. Therefore, the greenness of bond indices increases more quickly than the greenness of equity indices. All these factors show that the cost of implementing net zero investing with respect to traditional investing will be higher for equity portfolios than bond portfolios and the fixed-income market will benefit more quickly from the transition to a low-carbon economy.

In this dissertation, the cost is measured with respect to three risk dimensions: tracking risk, diversification risk and liquidity risk. We have put aside the question of financial performance, which we discussed in a previous publication (Laugel and Roncalli, 2022). The idea is not to reiterate what we have said. As shown by Pastor *et al.* (2021), the risk premium of green assets must be lower. Nevertheless, expected returns are different from actual returns, which depend on the investment flows and the supply/demand imbalance. Since we do not have a crystal ball, net zero portfolios may outperform or underperform business-as-usual portfolios. For instance, it is very interesting to notice that the investment universe of the greenest stocks from the transition viewpoint has behaved like a growth strategy in recent years. Indeed, we observe that these assets have been systematically overvalued, except during the Covid-19 crisis. Of course, this may change in the future. Investing in green assets could also be a low-risk or a quality strategy, or it could be correlated to momentum and value risk factors. In this case, predicting whether net zero investing has a positive or negative alpha is a pipe dream.

The final remark concerns the implementation of net zero investment policies. In this research, we have focused on the traditional top-down approach because we can easily obtain quantitative results. However, this is not the only solution. In particular, active management makes a lot of sense if we want to implement net zero investing. For instance, we have presented the core-satellite approach, which consists of the decarbonization dimension for the core investment and the transition dimension for the satellite strategy. This framework is easier to implement than the integrated optimization approach. Moreover, it allows control over the breakdown between the two dimensions, and the weight of the transition bucket to be progressively changed based on the greenness of the economy. Currently, net zero could be viewed as thematic investing because the universe of transition assets is small. But in the future, there will be no difference between net zero and core investing. If there is, that would mean that we have collectively failed to limit global warming.

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A Technical appendix

A.1 Notations

Table 38: Carbon risk measures

Symbol	Description
$\mathcal{CB}$	Carbon budget
$\mathcal{CE}$	Carbon emission
$\mathcal{CI}$	Carbon intensity
$\mathcal{CM}$	Carbon momentum
$\mathcal{R}$	Carbon reduction
$\boldsymbol{v}$	Carbon velocity
$\mathcal{SC}_1$	Scope 1
$\mathcal{SC}_2$	Scope 2
$\mathcal{SC}_3^{\text{up}}$	Upstream scope 3
$\mathcal{SC}_3^{\text{down}}$	Downstream scope 3
$\mathcal{SC}_3$	Scope 3 ($= \mathcal{SC}_3^{\text{up}} + \mathcal{SC}_3^{\text{down}}$)
$\mathcal{SC}_{1-2}$	Scope $1+2$
$\mathcal{SC}_{1-3}^{\text{up}}$	Upstream scope $1+2+3$ ($= \mathcal{SC}_1 + \mathcal{SC}_2 + \mathcal{SC}_3^{\text{up}}$)
$\mathcal{SC}_{1-3}$	Scope $1+2+3$

Table 39: Transition risk measures

Symbol	Description
$\mathcal{BI}$	Brown intensity
$\mathcal{GI}$	Green intensity
$\mathcal{GC}$	Green capex
$\mathcal{GM}$	Green momentum
$\mathcal{GRS}$	Green revenue share

A.2 Mathematical results

A.2.1 QP problem when there is a benchmark

Following Roncalli (2013) the excess return $R(x \mid b)$ of Portfolio x with respect to Benchmark b is the difference between the return of the portfolio and the return of the benchmark:

$$R(x \mid b) = R(x) - R(b) = (x-b)^\top R$$

It is easy to show that the expected excess return is equal to:

$$\mu(x \mid b) = \mathbb{E}\left[R(x \mid b)\right] = (x-b)^\top \mu$$

whereas the volatility of the tracking error is given by:

$$\sigma(x \mid b) = \sigma\left(R(x \mid b)\right) = \sqrt{(x-b)^\top \Sigma (x-b)}$$

The objective function is then:

$$\begin{aligned} f(x \mid b) &= \frac{1}{2}(x-b)^\top \Sigma (x-b) - \gamma (x-b)^\top \mu \\ &= \frac{1}{2}x^\top \Sigma x - x^\top (\gamma\mu + \Sigma b) + \left(\frac{1}{2}b^\top \Sigma b + \gamma b^\top \mu\right) \\ &= \frac{1}{2}x^\top Q x - x^\top R + C \end{aligned}$$

where C is a constant which does not depend on Portfolio x. We recognize a QP problem where $Q = \Sigma$ and $R = \gamma\mu + \Sigma b$.

A.2.2 Carbon momentum aggregation at the portfolio level

If we consider carbon momentums built on intensities, we recall that we have:

$$\mathcal{CM}_i^{\mathcal{L}ong}(t) = \frac{\hat{\beta}_{i,1}(t)}{\mathcal{CI}_i(t)} \tag{82}$$

where i is the issuer, $\mathcal{CI}_i(t)$ is the carbon intensity and $\hat{\beta}_{i,1}(t)$ is the slope of the trend model:

$$\widehat{\mathcal{CI}}_i(t) = \mathcal{CI}_i(t) + \hat{\beta}_{i,1}(t) \cdot (t - t_0)$$

Let $x = (x_1, \ldots, x_n)$ be the weights of the stocks that belong to the portfolio. Its carbon intensity is given by its weighted average:

$$\mathcal{CI}_x(t) = \sum_{i=1}^n x_i \cdot \mathcal{CI}_i(t) \tag{83}$$

It follows that:

$$\begin{aligned} \widehat{\mathcal{CI}}_x(t) &= \sum_{i=1}^n x_i \cdot \widehat{\mathcal{CI}}_i(t) \\ &= \sum_{i=1}^n x_i \cdot \mathcal{CI}_i(t_0) + (t-t_0)\sum_{i=1}^n x_i \cdot \hat{\beta}_{i,1}(t) \\ &= \mathcal{CI}_x(t_0) + \hat{\beta}_{x,1}(t) \cdot (t - t_0) \end{aligned}$$

where $\hat{\beta}_{x,1}(t) = \sum_{i=1}^{n} x_i \cdot \hat{\beta}_{i,1}(t)$. We deduce that:

$$\begin{aligned}
\boldsymbol{\mathcal{CM}}_x^{\mathcal{L}ong}(t) &= \frac{\hat{\beta}_{x,1}(t)}{\boldsymbol{\mathcal{CI}}_x(t)} && (84) \\
&= \frac{\sum_{i=1}^{n} x_i \cdot \hat{\beta}_{i,1}(t)}{\sum_{i=1}^{n} x_i \cdot \boldsymbol{\mathcal{CI}}_i(t)} && (85) \\
&= \frac{\sum_{i=1}^{n} x_i \cdot \boldsymbol{\mathcal{CI}}_i(t) \cdot \boldsymbol{\mathcal{CM}}_i^{\mathcal{L}ong}(t)}{\sum_{i=1}^{n} x_i \cdot \boldsymbol{\mathcal{CI}}_i(t)} && (86) \\
&= \sum_{i=1}^{n} \tilde{x}_i \cdot \boldsymbol{\mathcal{CM}}_i^{\mathcal{L}ong}(t) && (87)
\end{aligned}$$

where the adjusted weight $\tilde{x}_i$ is equal to:

$$\tilde{x}_i = \frac{x_i \cdot \boldsymbol{\mathcal{CI}}_i(t)}{\sum_{j=1}^{n} x_j \cdot \boldsymbol{\mathcal{CI}}_j(t)} \tag{88}$$

Remark 30. *In particular, we see that $\boldsymbol{\mathcal{CM}}_x^{\mathcal{L}ong}(t) \neq \sum_{i=1}^{n} x_i \cdot \boldsymbol{\mathcal{CM}}_i^{\mathcal{L}ong}(t)$. At the sector level, we aggregate the carbon momentum following the same method with the weights of each issuer in its respective sector.*

B Additional results

B.1 Tables

B.1.1 Data providers comparison

Table 40: Share (in %) of similar carbon emissions data between providers - $\mathcal{SC}_2$

	Provider	Provider 1	Provider 2	Provider 3
Equality	Provider 1	100	40	39
	Provider 2	40	100	14
	Provider 3	39	14	100

	Provider	Provider 1	Provider 2	Provider 3
10.00% difference	Provider	100	68	63
	Provider 2	68	100	30
	Provider 3	63	30	100

Table 41: Share (in %) of similar carbon emissions data between providers - $\mathcal{SC}_3$

	Provider	Provider 1	Provider 2	Provider 3
Equality	Provider 1	100	0	43
	Provider 2	0	100	0
	Provider 3	43	0	100

	Provider	Provider 1	Provider 2	Provider 3
10% difference	Provider 1	100	10	65
	Provider 2	10	100	8
	Provider 3	65	8	100

Table 42: Share (in %) of carbon emissions with a relative difference between providers lower than 10% - $\mathcal{SC}_3^{\mathrm{up}} and \mathcal{SC}_3^{\mathrm{down}}$

	Provider	Provider 1	Provider 2	Provider 3
Upstream	Provider 1	100	5	67
	Provider 2	5	100	4
	Provider 3	67	4	100

	Provider	Provider 1	Provider 2	Provider 3
Downstream	Provider 1	100	48	67
	Provider 2	48	100	32
	Provider 3	67	32	100

B.1.2 Carbon emissions

Table 43: Breakdown (in %) of carbon emissions in 2019

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_3^{\text{up}}$	$\mathcal{SC}_3^{\text{down}}$	$\mathcal{SC}_3$	$\mathcal{SC}_{1-3}$
Communication Services	0.1	5.1	0.8	1.5	0.2	0.4	0.5
Consumer Discretionary	1.7	9.7	2.9	14.1	10.2	10.8	9.1
Consumer Staples	2.3	6.7	2.9	18.6	1.6	4.4	4.1
Energy	15.0	8.5	14.0	14.1	40.1	36.0	31.2
Financials	0.7	1.8	0.9	2.6	1.8	2.0	1.7
Health Care	0.3	1.7	0.5	2.6	0.2	0.6	0.6
Industrials	10.2	8.9	10.0	15.6	24.2	22.8	20.0
Information Technology	0.6	6.8	1.5	4.9	2.3	2.7	2.5
Materials	29.8	40.7	31.4	20.2	13.5	14.6	18.2
Real Estate	0.3	2.8	0.6	1.1	1.0	1.0	0.9
Utilities	39.0	7.3	34.4	4.7	4.8	4.8	11.2
Total (in $GtCO_2e$)	15.1	2.6	17.6	10.3	53.7	64.0	81.6

Table 44: Distribution of carbon emissions in 2019

Sector	$\frac{\mathcal{SC}_2}{\mathcal{SC}_1}$	$\frac{\mathcal{SC}_3^{\text{up}}}{\mathcal{SC}_{1-2}}$	$\frac{\mathcal{SC}_3^{\text{down}}}{\mathcal{SC}_{1-2}}$	$\frac{\mathcal{SC}_3}{\mathcal{SC}_{1-2}}$	$\frac{\mathcal{SC}_{1-2}}{\mathcal{SC}_{1-3}}$	$\frac{\mathcal{SC}_3}{\mathcal{SC}_{1-3}}$
Communication Services	7.9	1.1	0.8	1.8	0.35	0.65
Consumer Discretionary	0.9	2.8	10.7	13.6	0.07	0.93
Consumer Staples	0.5	3.7	1.7	5.4	0.16	0.84
Energy	0.1	0.6	8.7	9.3	0.10	0.90
Financials	0.4	1.8	6.5	8.2	0.11	0.89
Health Care	1.1	3.2	1.3	4.5	0.18	0.82
Industrials	0.1	0.9	7.4	8.3	0.11	0.89
Information Technology	1.8	1.9	4.6	6.5	0.13	0.87
Materials	0.2	0.4	1.3	1.7	0.37	0.63
Real Estate	1.8	1.0	4.7	5.8	0.15	0.85
Utilities	0.0	0.1	0.4	0.5	0.67	0.33
Total	0.2	0.6	3.0	3.6	0.22	0.78

B.1.3 Portfolio decarbonization

We recall the equity portfolio optimization problem :

$$\begin{aligned} x^{\star}(\boldsymbol{\mathcal{R}}) &= \arg\min \frac{1}{2}(x-b)^{\top}\Sigma(x-b) \\ &\text{s.t.} \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}(x) \leq (1-\boldsymbol{\mathcal{R}}) \cdot \boldsymbol{\mathcal{CI}}(b) \\ x \in \Omega_1 \cap \Omega_2 \end{array} \right. \end{aligned} \tag{89}$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate and $\Omega = \Omega_1 \cap \Omega_2$ is a set of constraints.

We recall the bond portfolio optimization problem :

$$\begin{aligned} x^{\star}(\boldsymbol{\mathcal{R}}) &= \arg\min \varphi \underbrace{\sum_{s=1}^{n_{\mathcal{S}ector}} \left| \sum_{i \in s} (x_i - b_i) \cdot \mathrm{DTS}_i \right|}_{\text{DTS component}} + \underbrace{\frac{1}{2}\sum_{i \in b} |x_i - b_i|}_{\text{AS component}} \\ &\text{s.t.} \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}(x) \leq (1-\boldsymbol{\mathcal{R}}) \cdot \boldsymbol{\mathcal{CI}}(b) \\ x \in \Omega_1 \cap \Omega_2 \end{array} \right. \end{aligned} \tag{90}$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate, $\Omega_1 \cap \Omega_2$ is a set of constraints and φ is the trade-off coefficient between DTS and AS components.

Table 45: Sector allocation in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, scope $\boldsymbol{\mathcal{SC}}_{1-2}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.74	7.83	7.93	8.08	8.31	8.68	9.02
Consumer Discretionary	10.56	10.71	10.78	10.84	10.91	11.08	11.18	10.69
Consumer Staples	7.80	7.98	8.05	8.12	8.17	8.08	7.55	5.89
Energy	4.99	4.80	4.66	4.40	3.99	3.30	2.00	0.14
Financials	13.56	14.05	14.32	14.67	15.20	16.19	18.30	23.11
Health Care	14.15	14.40	14.53	14.68	14.90	15.21	15.73	16.02
Industrials	9.90	10.04	10.07	10.13	10.19	10.12	9.83	8.86
Information Technology	21.08	21.38	21.54	21.74	22.02	22.44	23.09	23.93
Materials	4.28	3.80	3.54	3.20	2.69	2.04	1.20	0.59
Real Estate	2.90	2.98	3.00	3.01	2.97	2.77	2.27	1.50
Utilities	3.21	2.11	1.70	1.28	0.88	0.45	0.19	0.24

Table 46: Sector allocation in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, scope $\boldsymbol{\mathcal{SC}}^{\mathrm{up}}_{1-3}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.95	8.28	8.81	9.61	10.50	10.88	0.18
Consumer Discretionary	10.56	10.76	10.77	10.67	10.35	9.44	6.86	0.00
Consumer Staples	7.80	7.44	6.99	6.17	4.94	3.12	0.93	0.24
Energy	4.99	4.55	4.06	3.36	2.28	1.00	0.00	0.00
Financials	13.56	14.90	15.99	17.86	21.09	26.04	37.75	81.71
Health Care	14.15	14.69	15.02	15.39	15.61	14.87	10.74	3.98
Industrials	9.90	9.94	9.76	9.07	7.55	6.30	4.74	2.85
Information Technology	21.08	21.73	22.18	22.78	23.48	24.07	24.41	9.55
Materials	4.28	3.16	2.43	1.58	0.86	0.43	0.22	0.17
Real Estate	2.90	3.21	3.39	3.60	3.80	3.83	3.02	0.94
Utilities	3.21	1.67	1.12	0.71	0.43	0.41	0.45	0.38

Table 47: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\boldsymbol{\mathcal{SC}}_1$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Consumer Discretionary	5.97	0.00	0.00	0.09	0.09	0.09	0.00	0.02
Consumer Staples	6.04	0.00	0.00	0.00	0.00	0.00	0.00	−0.01
Energy	6.49	0.00	0.00	0.02	0.02	0.38	0.70	−0.12
Financials	33.91	0.45	0.65	1.13	1.45	1.15	1.85	3.02
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industrials	8.92	0.00	0.00	0.00	0.00	−0.02	−0.22	−0.42
Information Technology	5.57	0.11	0.11	0.00	0.32	0.02	0.00	0.00
Materials	3.44	−0.10	−0.12	−0.13	−0.16	−0.20	−0.68	−0.92
Real Estate	4.76	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01
Utilities	10.06	−0.47	−0.64	−1.11	−1.72	−1.43	−1.63	−1.58

Table 48: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\boldsymbol{\mathcal{SC}}_{1-2}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$ 30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Consumer Discretionary	5.97	0.00	−0.04	−0.04	−0.04	−0.06	−0.08	−0.08
Consumer Staples	6.04	0.00	0.00	0.00	0.00	0.00	−0.01	−0.07
Energy	6.49	0.00	0.00	−0.10	−0.12	−0.06	−0.21	−2.88
Financials	33.91	0.72	1.14	1.84	2.22	2.35	3.06	5.28
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industrials	8.92	0.00	0.00	0.00	0.00	0.03	0.02	0.73
Information Technology	5.57	0.00	0.00	0.00	0.00	0.00	0.00	−0.05
Materials	3.44	−0.11	−0.16	−0.17	−0.19	−0.43	−0.85	−1.40
Real Estate	4.76	0.00	0.00	0.00	0.00	0.00	−0.06	−0.12
Utilities	10.06	−0.61	−0.95	−1.53	−1.87	−1.83	−1.87	−1.41

Table 49: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$ 30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	−0.04	−0.03	0.02	−0.05	0.00	−0.17	−1.29
Consumer Discretionary	5.97	0.00	−0.02	−0.03	−0.04	−0.06	−0.18	−2.81
Consumer Staples	6.04	0.00	−0.01	−0.08	−0.31	−0.81	−2.41	−3.72
Energy	6.49	0.00	−0.07	0.21	0.53	1.02	1.85	2.21
Financials	33.91	1.43	2.72	2.97	4.39	5.38	8.10	14.88
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	−0.07	−1.96
Industrials	8.92	0.00	0.00	0.01	−0.19	−0.29	−0.76	4.01
Information Technology	5.57	0.00	0.00	0.00	0.00	0.00	−0.17	−2.17
Materials	3.44	−0.09	−0.14	−0.17	−0.59	−0.87	−1.04	−1.22
Real Estate	4.76	0.00	0.00	0.00	−0.01	−0.06	−0.13	−2.56
Utilities	10.06	−1.30	−2.46	−2.92	−3.74	−4.31	−5.03	−5.39

Table 50: Contribution to yield variation in bps (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Rating								
AAA–AA	33	1	0	0	0	0	0	0
A	160	0	−1	0	1	0	3	4
BBB	229	−1	−1	−2	−7	−8	−12	−26
Duration								
0Y–2Y	41	1	1	1	0	2	3	1
2Y–5Y	148	−1	−2	−3	−5	−8	−11	−23
5Y–7Y	67	0	−1	−1	−1	−1	1	5
7Y–10Y	60	0	0	−1	−1	−2	−1	−2
10Y+	107	1	1	1	0	0	−1	−3
Sector								
Communication Services	32	0	0	0	0	0	0	0
Consumer Discretionary	24	0	0	0	0	−1	−4	−6
Consumer Staples	25	0	0	0	0	0	−1	−4
Energy	30	−4	−7	−9	−9	−11	−14	−14
Financials	138	4	6	8	9	15	22	19
Health Care	31	0	0	0	0	0	0	0
Industrials	37	1	1	2	3	3	3	9
Information Technology	24	0	0	0	0	0	0	−1
Materials	16	0	−1	−1	−1	−3	−5	−7
Real Estate	24	0	0	0	0	0	0	−3
Utilities	43	−1	−1	−2	−8	−10	−11	−15

Table 51: Weight (in %) and yield (in bps) variations of the Financials sector (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

		Index	Reduction rate $\mathcal{R}$ 30%	40%	50%	60%	70%	80%	90%
Weight									
AAA–AA	0Y–2Y	0.8	0.0	0.2	0.7	1.3	1.4	2.5	4.2
	2Y–5Y	1.6	0.4	0.8	0.6	0.1	0.3	−0.2	−0.9
	5Y–7Y	0.4	0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.2
	7Y–10Y	0.2	0.0	0.0	0.0	0.0	0.0	0.0	−0.1
	10Y+	0.4	0.0	0.0	0.0	0.0	0.0	0.0	−0.3
A	0Y–2Y	3.6	0.0	0.0	−0.1	−0.1	−0.1	−0.1	0.4
	2Y–5Y	9.7	0.0	0.1	0.2	1.0	1.5	2.9	0.8
	5Y–7Y	2.8	0.5	0.9	1.0	0.6	0.6	0.0	2.4
	7Y–10Y	2.4	0.0	0.0	0.0	0.0	0.0	−0.1	3.0
	10Y+	2.0	0.0	0.0	0.0	0.0	0.0	0.0	−0.9
BBB	0Y–2Y	1.7	0.4	0.4	0.2	−0.1	0.6	0.6	−0.6
	2Y–5Y	4.9	−0.3	−0.3	−0.4	−0.4	−0.4	−0.5	−1.8
	5Y–7Y	1.3	−0.1	−0.1	−0.1	−0.1	−0.1	0.2	−0.1
	7Y–10Y	1.1	0.0	0.0	−0.1	−0.1	−0.1	−0.2	−0.4
	10Y+	0.9	0.0	0.0	0.0	0.0	0.0	−0.1	−0.4
Yield									
AAA–AA	0Y–2Y	2	0	1	3	5	5	9	15
	2Y–5Y	5	2	3	3	1	1	0	−3
	5Y–7Y	1	0	0	0	0	0	0	−1
	7Y–10Y	1	0	0	0	0	0	0	0
	10Y+	2	0	0	0	0	0	0	−1
A	0Y–2Y	11	0	0	0	0	0	0	2
	2Y–5Y	38	0	0	1	4	7	13	6
	5Y–7Y	12	2	2	4	2	2	0	12
	7Y–10Y	11	0	0	0	0	0	0	9
	10Y+	10	0	0	0	0	0	0	−4
BBB	0Y–2Y	6	2	2	1	0	4	4	−2
	2Y–5Y	22	−1	−1	−2	−2	−2	−2	−8
	5Y–7Y	7	0	0	−1	−1	−1	2	1
	7Y–10Y	6	0	0	0	0	0	−1	−2
	10Y+	5	0	0	0	0	0	0	−2

Table 52: Weight (in %) and yield (in bps) variations of the Utilities sector (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

		Index	Reduction rate $\mathcal{R}$						
			30%	40%	50%	60%	70%	80%	90%
Weight									
AAA–AA	0Y–2Y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2Y–5Y	0.1	0.0	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
	5Y–7Y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	7Y–10Y	0.1	0.0	0.0	0.0	0.0	−0.1	−0.1	−0.1
	10Y+	0.1	0.0	0.0	0.0	0.0	−0.1	−0.1	−0.1
A	0Y–2Y	0.3	0.0	0.0	0.0	−0.2	−0.2	−0.2	−0.2
	2Y–5Y	0.7	0.0	0.0	−0.1	−0.5	−0.5	−0.6	−0.6
	5Y–7Y	0.4	0.0	0.0	−0.1	−0.2	−0.3	−0.3	−0.3
	7Y–10Y	0.6	0.3	0.6	0.8	1.2	1.9	2.4	−0.6
	10Y+	1.7	0.0	−0.1	0.0	0.5	0.2	1.9	4.8
BBB	0Y–2Y	0.6	0.0	−0.1	−0.1	−0.2	−0.3	−0.5	−0.5
	2Y–5Y	2.0	−0.1	−0.2	−0.3	−0.9	−1.2	−1.7	−1.9
	5Y–7Y	1.1	0.0	−0.1	−0.2	−0.4	−0.7	−1.0	−1.1
	7Y–10Y	0.9	0.0	0.0	−0.1	−0.2	−0.5	−0.7	−0.9
	10Y+	1.3	0.0	0.0	−0.2	−0.4	−0.1	−1.1	−1.3
Yield									
AAA–AA	0Y–2Y	0	0	0	0	0	0	0	0
	2Y–5Y	0	0	0	0	0	0	0	0
	5Y–7Y	0	0	0	0	0	0	0	0
	7Y–10Y	0	0	0	0	0	0	0	0
	10Y+	1	0	0	0	0	−1	−1	−1
A	0Y–2Y	1	0	0	0	−1	−1	−1	−1
	2Y–5Y	2	0	0	0	−2	−2	−2	−2
	5Y–7Y	1	0	0	0	−1	−1	−1	−1
	7Y–10Y	3	1	2	3	5	7	10	−2
	10Y+	8	0	−1	0	1	0	7	18
BBB	0Y–2Y	2	0	0	0	−1	−1	−2	−2
	2Y–5Y	8	0	−1	−1	−4	−5	−7	−8
	5Y–7Y	5	0	0	−1	−2	−3	−4	−5
	7Y–10Y	4	0	0	0	−1	−2	−3	−4
	10Y+	7	0	0	−1	−2	0	−5	−7

Table 53: Green intensity in % (Global Corp., Jun. 2022), average sector data applied for missing green data

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	3.82	4.03	4.18	4.30	4.53	4.90	5.45	6.14
$\mathcal{SC}_{1-2}$		3.77	3.72	3.69	3.62	3.64	3.62	3.23
$\mathcal{SC}_{1-3}^{\text{up}}$		3.89	3.81	4.09	4.13	4.12	3.55	2.01
$\mathcal{SC}_{1-3}$		3.90	4.06	4.29	4.98	5.38	5.95	5.61

B.1.4 Marginal impact and integrated approach

We recall the integrated approach optimization problem for equity :

$$\begin{array}{rcl} x^{\star}(t) & = & \arg\min \frac{1}{2}\left(x-b(t)\right)^{\top}\Sigma(t)\left(x-b(t)\right) \\ & & \text{s.t.} \quad \left\{ \begin{array}{ll} \boldsymbol{\mathcal{CI}}(t,x) \leq \left(1-\boldsymbol{\mathcal{R}}(t_0,t)\right)\cdot\boldsymbol{\mathcal{CI}}\left(t_0,b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ x \in \Omega_{\boldsymbol{\mathcal{T}}ransition}(t) & \longleftarrow \text{Transition} \\ x \in \Omega_1 \cap \Omega_2(t) & \end{array} \right. \end{array} \tag{91}$$

where the decarbonization dimension is defined by using the usual constraint $\boldsymbol{\mathcal{CI}}(t,x) \leq \left(1-\boldsymbol{\mathcal{R}}(t_0,t)\right)\cdot \boldsymbol{\mathcal{CI}}\left(t_0,b(t_0)\right)$ and the transition dimension is specified by the set of constraints $\Omega_{\boldsymbol{\mathcal{T}}ransition}(t)$. We assume that:

$$x \in \Omega_{\boldsymbol{\mathcal{T}}ransition}(t) \Leftrightarrow \left\{ \begin{array}{ll} \boldsymbol{\mathcal{GI}}(t,x) \geq \left(1+\boldsymbol{\mathcal{G}}(t)\right)\cdot\boldsymbol{\mathcal{GI}}\left(t_0,b(t_0)\right) & \longleftarrow \text{Greenness} \\ \boldsymbol{\mathcal{CM}}^{\mathcal{L}ong}(t,x) \leq \boldsymbol{\mathcal{CM}}^{\star} & \longleftarrow \text{Momentum} \end{array} \right. \tag{92}$$

The problem is extended to bond portfolios by using the DTS and AS components.

Table 54: Additional tracking error cost in bps of the greenness constraint (MSCI World, Jun. 2022, $\mathcal{C}_3(0,10,2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\boldsymbol{\mathcal{G}} = 0\%$				
$\boldsymbol{\mathcal{SC}}_1$	0	0	0	0	0	0	0	0	0
$\boldsymbol{\mathcal{SC}}_{1-2}$	0	0	0	0	0	0	0	0	1
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	0	0	0	0	0	4			
$\boldsymbol{\mathcal{SC}}_{1-3}$	0	0	0	0	0	0	1	6	
					$\boldsymbol{\mathcal{G}} = 100\%$				
$\boldsymbol{\mathcal{SC}}_1$	24	23	22	22	19	16	14	13	13
$\boldsymbol{\mathcal{SC}}_{1-2}$	23	22	22	21	19	20	21	30	51
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	18	18	18	18	23	83			
$\boldsymbol{\mathcal{SC}}_{1-3}$	18	17	16	16	15	16	24	133	
					$\boldsymbol{\mathcal{G}} = 200\%$				
$\boldsymbol{\mathcal{SC}}_1$	69	69	68	67	64	61	59	58	62
$\boldsymbol{\mathcal{SC}}_{1-2}$	69	69	68	68	71	78	93	135	233
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	67	68	70	72	112				
$\boldsymbol{\mathcal{SC}}_{1-3}$	62	62	61	61	64	73	137		

Table 55: Additional tracking error cost in bps of a global momentum threshold approach (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{CM}^{\star} = -5\%$				
$\mathcal{SC}_{1}$	11	11	11	11	9	8	8	7	6
$\mathcal{SC}_{1-2}$	10	9	9	9	6	6	4	2	1
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	5	4	3	3	2	1			
$\mathcal{SC}_{1-3}$	4	4	3	3	3	2	3	7	
					$\mathcal{CM}^{\star} = -7\%$				
$\mathcal{SC}_{1}$	23	23	23	23	22	21	19	19	20
$\mathcal{SC}_{1-2}$	21	21	20	20	19	16	14	16	13
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	14	13	12	10	6	8			
$\mathcal{SC}_{1-3}$	11	10	10	9	9	9	8	18	

Table 56: Additional tracking error cost in bps of a momentum-based exclusion approach (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{CM}^{+} = 0\%$				
$\mathcal{SC}_{1}$	124	122	122	120	114	107	99	89	81
$\mathcal{SC}_{1-2}$	121	120	118	117	108	96	80	64	41
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	109	105	101	98	74	44			
$\mathcal{SC}_{1-3}$	112	109	107	105	94	84	76	80	
					$\mathcal{CM}^{+} = 10\%$				
$\mathcal{SC}_{1}$	2	2	2	1	1	1	0	0	0
$\mathcal{SC}_{1-2}$	2	1	1	1	1	0	0	0	0
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	1	1	1	1	0	0			
$\mathcal{SC}_{1-3}$	1	1	1	1	1	0	0	0	0

Table 57: Active share (in %) between the decarbonized and net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$				
$\mathcal{SC}_{1}$	11.6	11.6	11.6	11.5	11.4	11.5	11.6	11.5	11.3
$\mathcal{SC}_{1-2}$	11.6	11.6	11.6	11.6	12.0	12.2	12.1	12.1	12.3
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	11.3	11.4	11.4	11.4	11.4	12.6	17.8	37.6	
$\mathcal{SC}_{1-3}$	11.4	11.3	11.3	11.3	11.1	11.3	12.9	16.2	21.6
					Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^{\star} = -7\%$				
$\mathcal{SC}_{1}$	22.1	22.0	22.0	21.9	21.8	21.8	21.9	21.9	21.6
$\mathcal{SC}_{1-2}$	22.0	22.0	22.0	22.0	22.4	22.7	22.4	22.5	22.9
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	21.5	21.4	21.3	21.3	21.3	23.4	29.8	56.0	
$\mathcal{SC}_{1-3}$	21.7	21.6	21.5	21.4	21.0	21.0	24.8	30.3	36.0

Table 58: Active share (in %) between the decarbonized and net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	13.3	13.3	13.2	13.2	13.2	13.1	14.1	15.4	17.7
$\mathcal{SC}_{1-2}$	13.3	13.3	13.4	13.4	13.6	15.2	16.6	19.7	23.2
$\mathcal{SC}_{1-3}^{\text{up}}$	12.9	12.7	12.8	13.2	16.0	30.5			
$\mathcal{SC}_{1-3}$	12.8	12.7	12.7	12.8	13.2	14.6	19.2	46.0	
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	33.1	33.2	33.3	33.5	34.6	35.9	36.5	39.8	45.5
$\mathcal{SC}_{1-2}$	33.7	34.1	34.5	34.9	36.7	38.0	44.1	53.5	57.9
$\mathcal{SC}_{1-3}^{\text{up}}$	33.7	34.3	35.0	35.3	44.1				
$\mathcal{SC}_{1-3}$	31.6	31.7	31.9	32.1	33.6	39.1	56.2		

Table 59: Additional active share cost in % when we control the green intensity (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 0\%$								
$\mathcal{SC}_1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathcal{SC}_{1-2}$	0.0	0.0	0.0	0.1	0.1	0.2	0.1	-0.2	-0.3
$\mathcal{SC}_{1-3}^{\text{up}}$	0.0	0.1	0.1	0.1	0.2	0.0	-0.2	-0.1	0.4
$\mathcal{SC}_{1-3}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	$\mathcal{G} = 100\%$								
$\mathcal{SC}_1$	2.2	1.9	1.7	1.4	0.2	0.2	0.2	0.0	-0.2
$\mathcal{SC}_{1-2}$	2.0	1.7	1.4	1.1	0.4	0.4	0.2	-0.0	-0.1
$\mathcal{SC}_{1-3}^{\text{up}}$	1.5	1.0	0.6	0.4	0.5	0.5	0.5	0.4	2.6
$\mathcal{SC}_{1-3}$	0.9	0.6	0.4	0.4	0.3	0.4	0.5	0.3	-0.7
	$\mathcal{G} = 200\%$								
$\mathcal{SC}_1$	5.6	5.3	5.1	4.8	3.1	1.9	0.7	0.2	0.0
$\mathcal{SC}_{1-2}$	5.4	5.2	4.8	4.4	2.6	1.5	0.9	0.4	0.3
$\mathcal{SC}_{1-3}^{\text{up}}$	4.9	4.3	3.6	3.0	1.3	1.3	1.6	1.6	6.5
$\mathcal{SC}_{1-3}$	4.2	3.5	2.9	2.4	0.8	0.7	0.9	0.7	-0.1

Table 60: Additional active share cost in % of a momentum exclusion approach (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	20.0	20.1	20.3	20.4	20.5	20.1	18.5	17.5	16.6
$\mathcal{SC}_{1-2}$	19.8	19.9	20.0	20.1	19.6	18.4	16.4	15.0	13.4
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	19.4	19.4	19.4	19.2	16.8	13.6	11.4	9.5	3.0
$\mathcal{SC}_{1-3}$	19.1	18.6	18.2	17.8	16.2	14.4	13.1	12.5	12.2
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.4	0.4	0.4	0.5	1.0	1.0	0.9	0.9	0.8
$\mathcal{SC}_{1-2}$	0.3	0.3	0.4	0.5	0.8	0.6	0.6	0.4	0.2
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.4	0.5	0.7	0.9	0.6	0.4	0.2	0.2	0.0
$\mathcal{SC}_{1-3}$	0.7	1.0	1.1	1.1	1.0	0.8	0.6	0.5	0.3

Table 61: Additional DTS cost in bps of a momentum exclusion approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	4.62	4.66	4.67	4.70	4.05	4.13	3.72	3.59	3.52
$\mathcal{SC}_{1-2}$	4.51	4.39	4.39	4.14	3.65	3.58	2.93	2.61	1.54
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	3.81	3.71	3.55	3.46	2.89	2.24	1.67	0.38	0.00
$\mathcal{SC}_{1-3}$	3.84	3.70	3.72	3.65	3.05	2.80	2.47	2.13	2.35
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.20	0.30	0.29	0.39	0.35	0.34	0.27	0.27	0.27
$\mathcal{SC}_{1-2}$	0.28	0.22	0.28	0.29	0.21	0.23	0.18	0.05	0.05
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.35	0.26	0.32	0.32	0.16	0.05	0.03	0.00	0.00
$\mathcal{SC}_{1-3}$	0.36	0.38	0.46	0.32	0.16	0.24	0.23	0.12	0.04

Table 62: Carbon momentum difference $\Delta\mathcal{CM}(t)$ in % of a momentum exclusion approach (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	−3.3	−3.3	−3.3	−3.2	−2.9	−2.8	−2.2	−2.4	−1.8
$\mathcal{SC}_{1-2}$	−2.3	−2.3	−2.2	−2.0	−1.6	−1.4	−1.2	−0.9	−0.6
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	−2.3	−2.3	−2.0	−1.8	−1.5	−2.1	−1.8	−2.8	−4.8
$\mathcal{SC}_{1-3}$	−2.1	−2.1	−2.2	−2.2	−1.6	−1.3	−1.4	−0.6	−0.8
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	−0.1	0.1	0.3	0.5	1.3	1.5	1.3	1.7	1.9
$\mathcal{SC}_{1-2}$	−0.1	0.1	0.5	0.6	1.2	0.8	0.3	0.6	0.8
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	−0.1	0.3	0.4	0.6	0.3	−0.0	−0.4	−1.7	−4.4
$\mathcal{SC}_{1-3}$	−0.5	−0.5	−0.7	−1.0	0.3	0.3	0.2	1.1	1.0

Table 63: Carbon momentum difference $\Delta\mathcal{CM}(t)$ in % of a momentum exclusion approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	−2.9	−2.9	−2.8	−2.8	−2.8	−2.3	−2.4	−1.9	−1.6
$\mathcal{SC}_{1-2}$	−1.7	−1.7	−1.6	−1.6	−1.4	−1.3	−0.9	−0.6	−0.5
$\mathcal{SC}_{1-3}^{\text{up}}$	−1.6	−1.7	−1.6	−1.6	−2.0	−1.8	−2.7	−4.6	−5.2
$\mathcal{SC}_{1-3}$	−2.2	−2.3	−1.9	−1.6	−1.3	−1.5	−0.6	−0.7	−1.0
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.8	0.9	1.0	1.2	1.5	1.3	1.6	1.9	1.9
$\mathcal{SC}_{1-2}$	0.8	0.8	1.0	1.1	0.8	0.3	0.6	0.8	0.5
$\mathcal{SC}_{1-3}^{\text{up}}$	0.8	0.6	0.5	0.4	−0.1	−0.2	−1.7	−4.0	−5.1
$\mathcal{SC}_{1-3}$	−0.7	−0.2	−0.0	0.2	0.3	0.2	1.1	1.1	1.0

Table 64: Additional Active share cost in % of a global momentum threshold approach (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	0.7	0.6	0.6	0.6	0.7	0.7	0.2	0.2	0.2
$\mathcal{SC}_{1-2}$	0.6	0.5	0.5	0.5	0.6	0.3	0.1	0.2	0.2
$\mathcal{SC}_{1-3}^{\text{up}}$	0.4	0.4	0.4	0.4	0.3	0.2	0.2	0.1	1.0
$\mathcal{SC}_{1-3}$	0.5	0.4	0.3	0.2	0.4	0.4	0.4	0.7	1.0
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	1.7	1.6	1.6	1.5	1.2	1.4	0.7	0.4	0.5
$\mathcal{SC}_{1-2}$	1.6	1.5	1.4	1.3	1.1	1.0	0.4	0.4	0.4
$\mathcal{SC}_{1-3}^{\text{up}}$	1.3	1.1	1.0	0.9	0.8	0.5	0.5	0.4	0.0
$\mathcal{SC}_{1-3}$	1.4	1.2	0.9	0.7	0.7	0.7	0.9	1.2	1.7

Table 65: Additional Active share cost in % of the constraint $\Omega_{\mathcal{G}reen\mathcal{W}ash}$ (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4
$\mathcal{SC}_{1-2}$	0.2	0.2	0.2	0.2	0.2	0.2	0.5	1.1	2.6
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.3	0.3	0.2	0.3	0.3	1.0	2.3	9.6	−1.4
$\mathcal{SC}_{1-3}$	0.4	0.6	0.7	0.6	0.6	1.1	1.8	3.1	7.1
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.3
$\mathcal{SC}_{1-2}$	0.2	0.2	0.2	0.2	0.1	0.1	0.5	1.0	2.4
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.2	0.2	0.2	0.2	0.3	1.0	2.3	8.8	1.8
$\mathcal{SC}_{1-3}$	0.4	0.5	0.6	0.6	0.5	1.0	1.8	3.0	7.0

B.2 Figures

B.2.1 Net zero scenarios

Figure 34: Estimated value $\Delta\boldsymbol{\mathcal{R}}^{\star}(2020, t)$ (in %) from the IEA NZE scenario — $g_Y = 6\%$

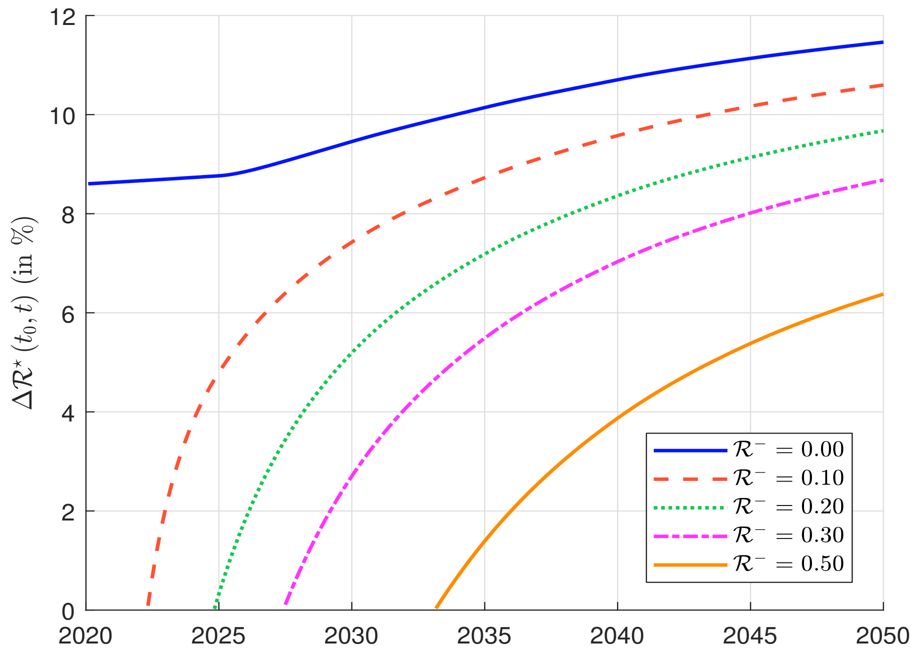


Figure 35: Estimated value $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) from the IEA NZE scenario — $g_Y = 6\%$

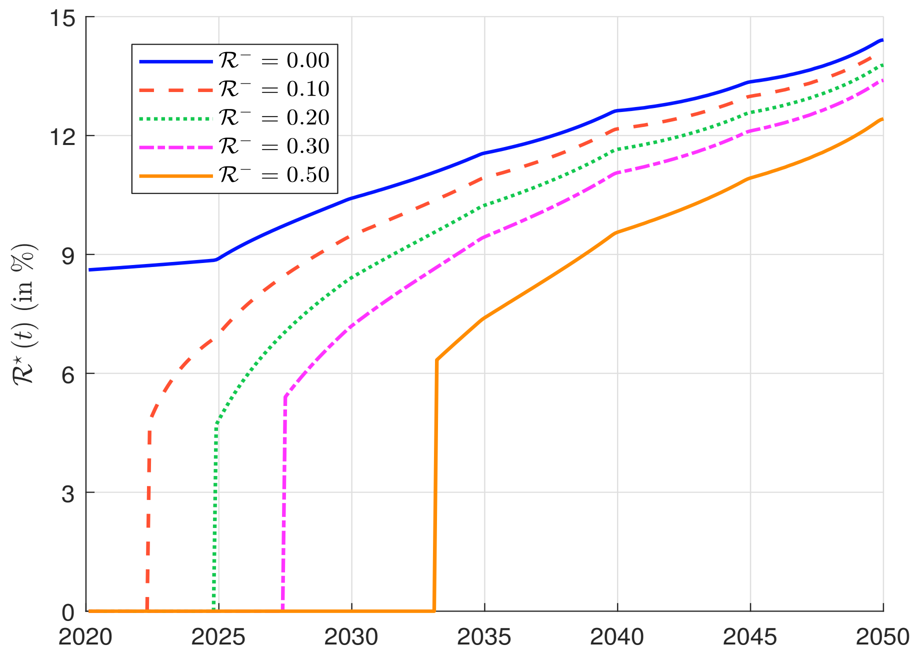


Figure 36: Relationship between $\Delta\boldsymbol{\mathcal{R}}^{\star}(2020, t)$ and $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) — $g_Y = 3\%$, $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}} = 0$

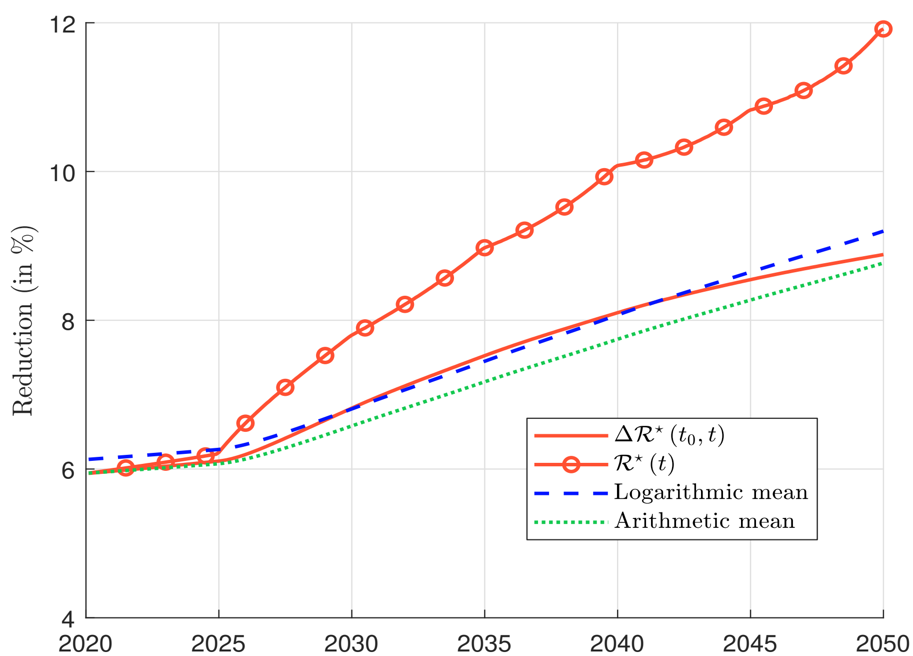


Figure 37: Relationship between $\Delta\boldsymbol{\mathcal{R}}^{\star}(2020, t)$ and $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) — $g_Y = 3\%$, $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}} = 0$

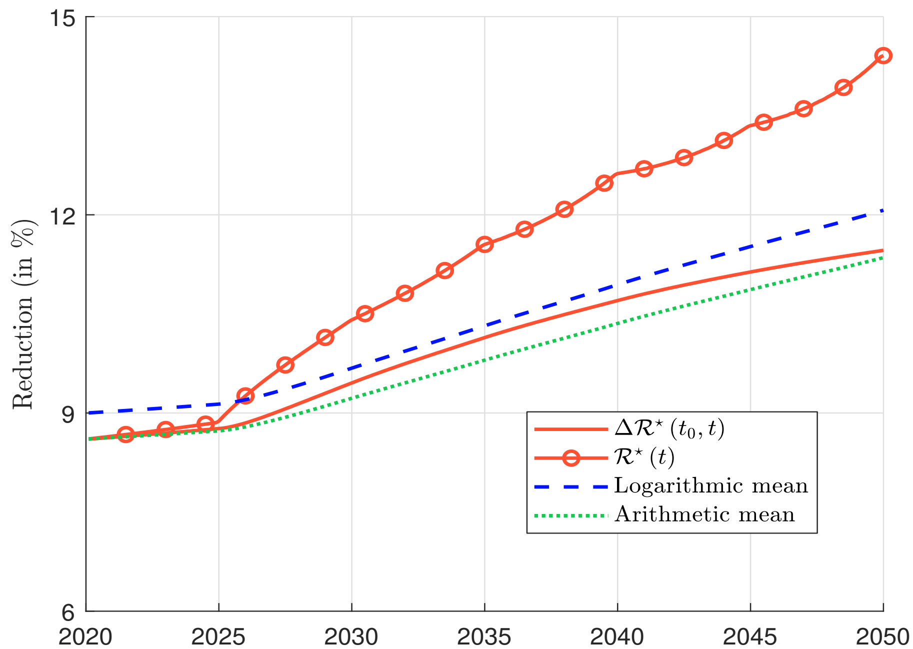


Figure 38: Fitted CTB and PAB decarbonization pathways

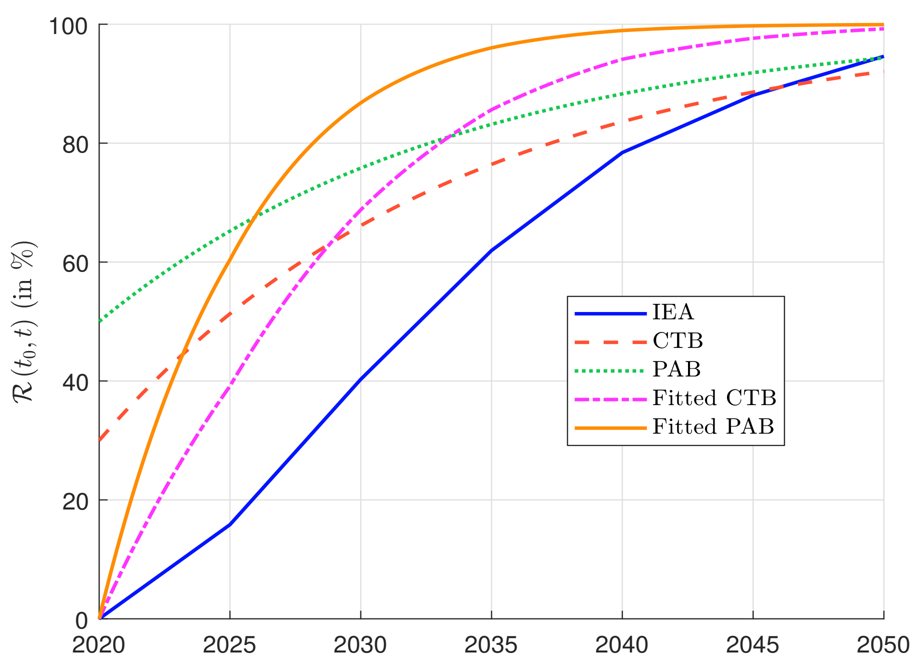
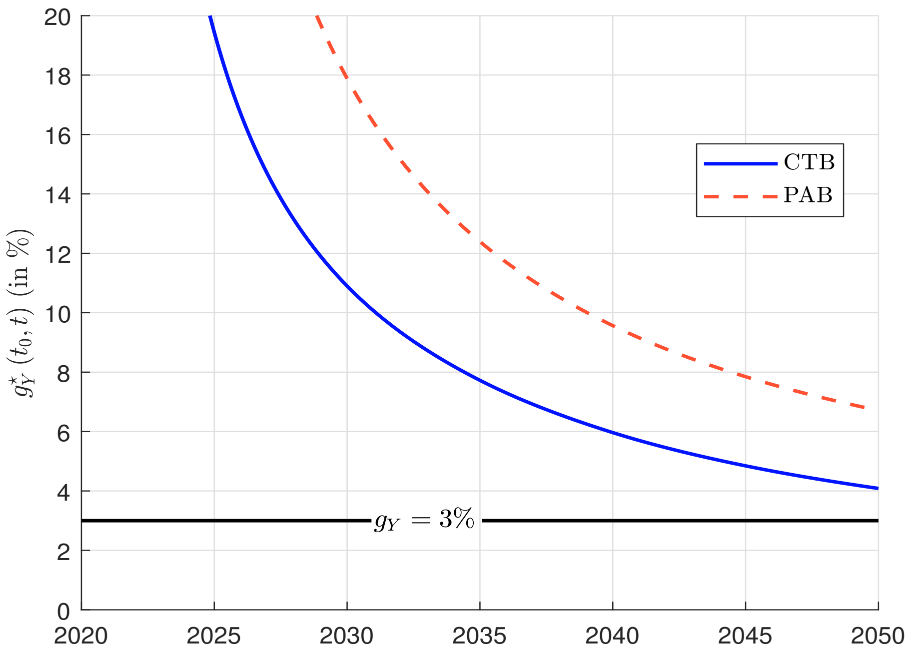


Figure 39: Calibrated growth rate $g_Y^{\star}(t_0, t)$



B.2.2 Self-decarbonization

Figure 40: Sequential versus self-decarbonization (Case #1)

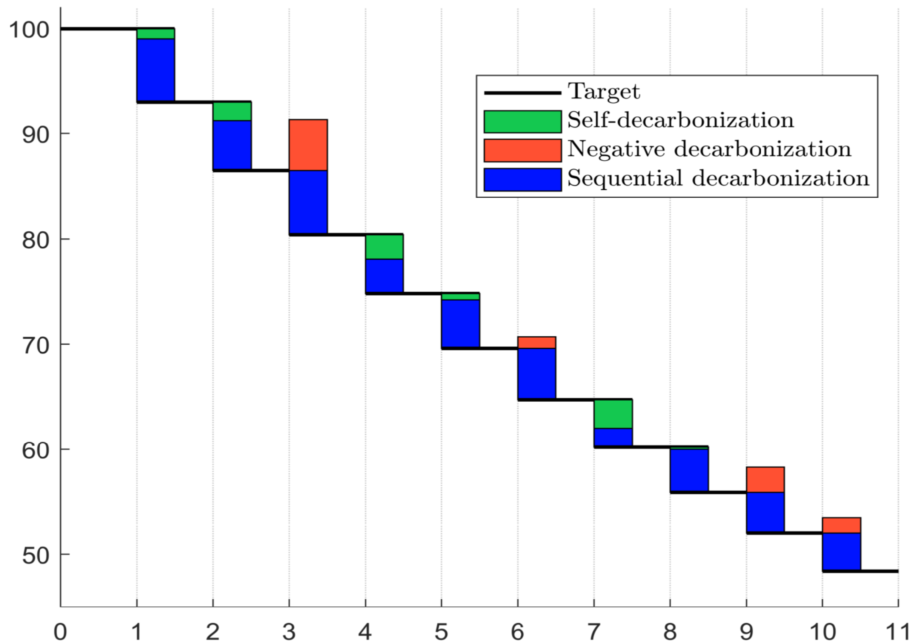
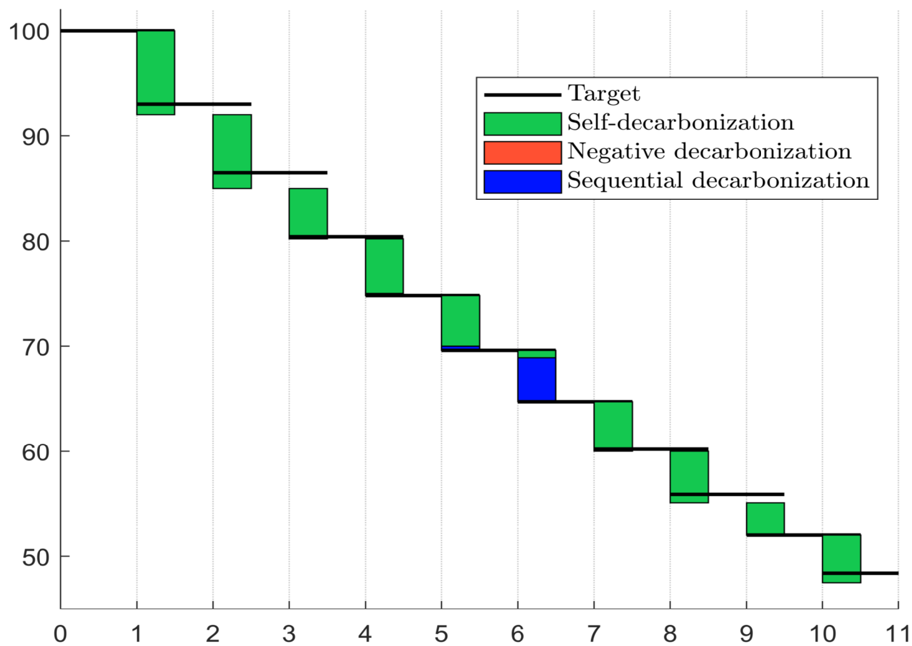


Figure 41: Sequential versus self-decarbonization (Case #2)



B.2.3 Carbon intensities

Figure 42: Boxplot of carbon intensity per sector (MSCI World, Jun. 2022, scope $\mathcal{SC}_{1-2}$)

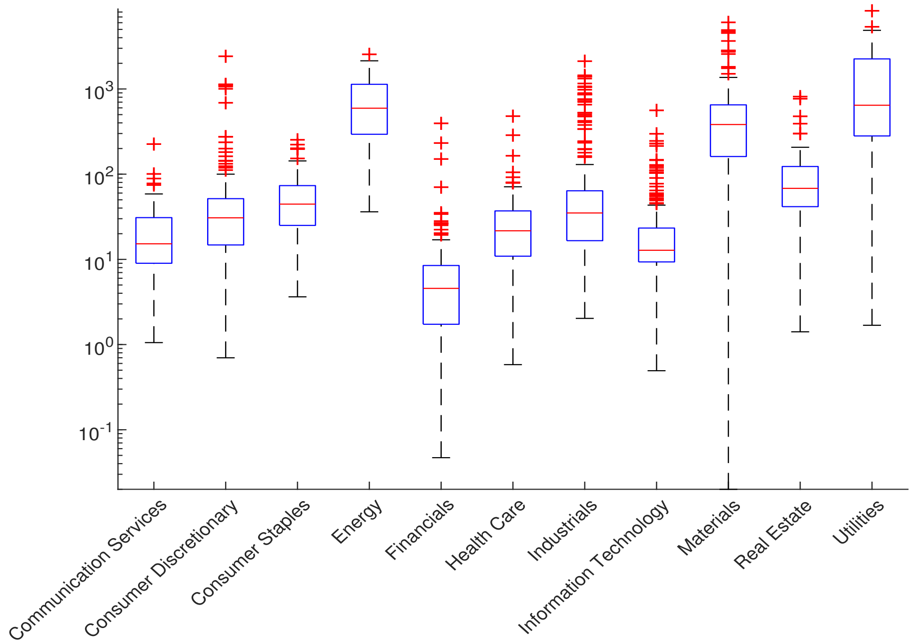
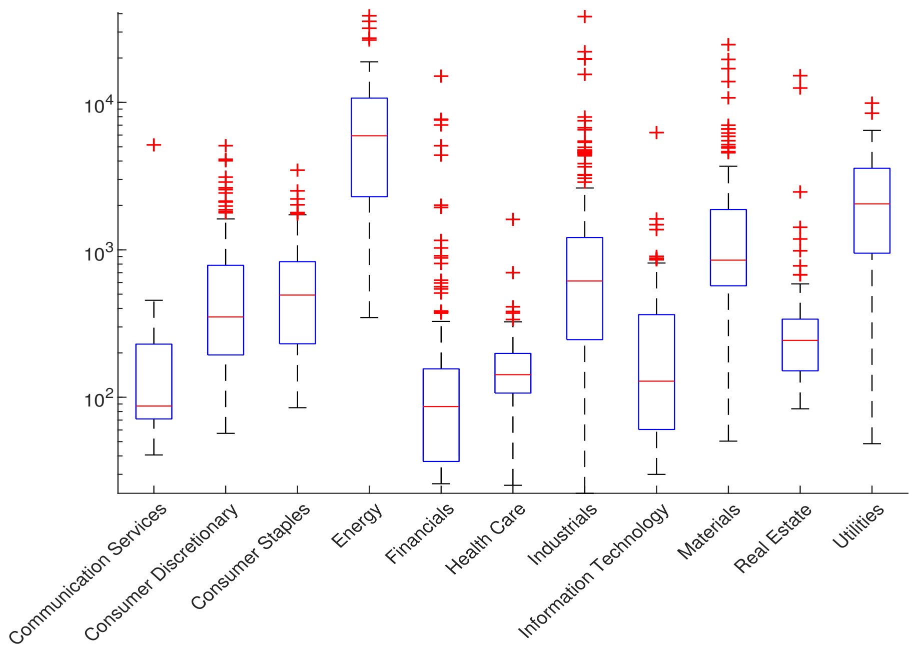


Figure 43: Boxplot of carbon intensity per sector (MSCI World, Jun. 2022, scope $\mathcal{SC}_{1-3}$)



B.2.4 Integrated approach tracking errors

Figure 44: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, CTB)

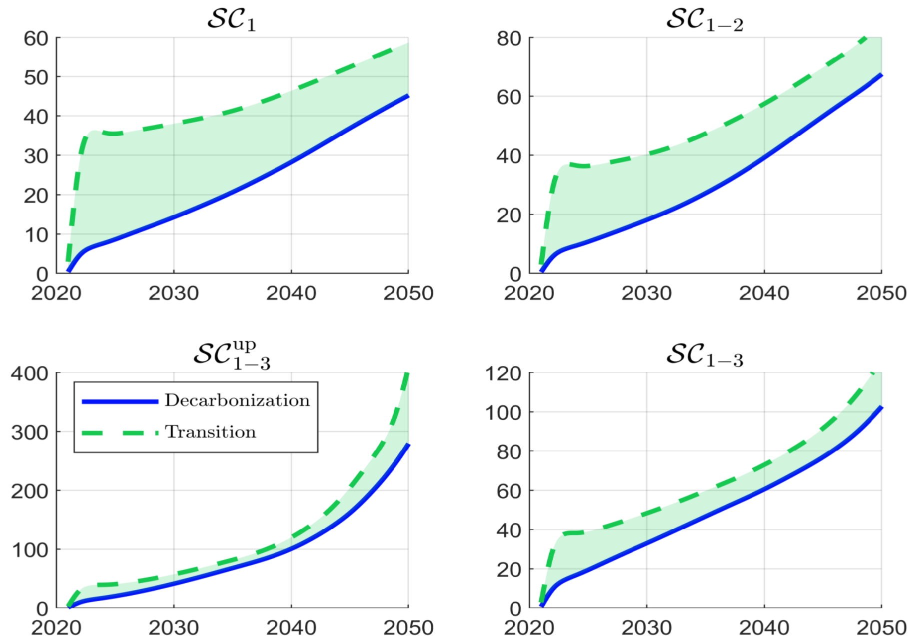


Figure 45: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 200\%$, $\boldsymbol{\mathcal{CM}}^\star = -7\%$, CTB)

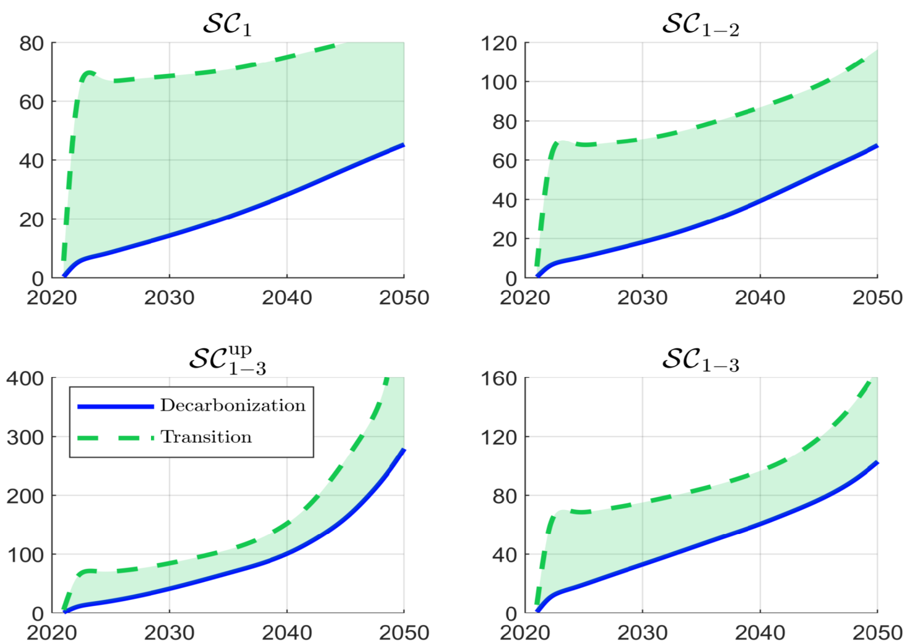


Figure 46: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, CTB)

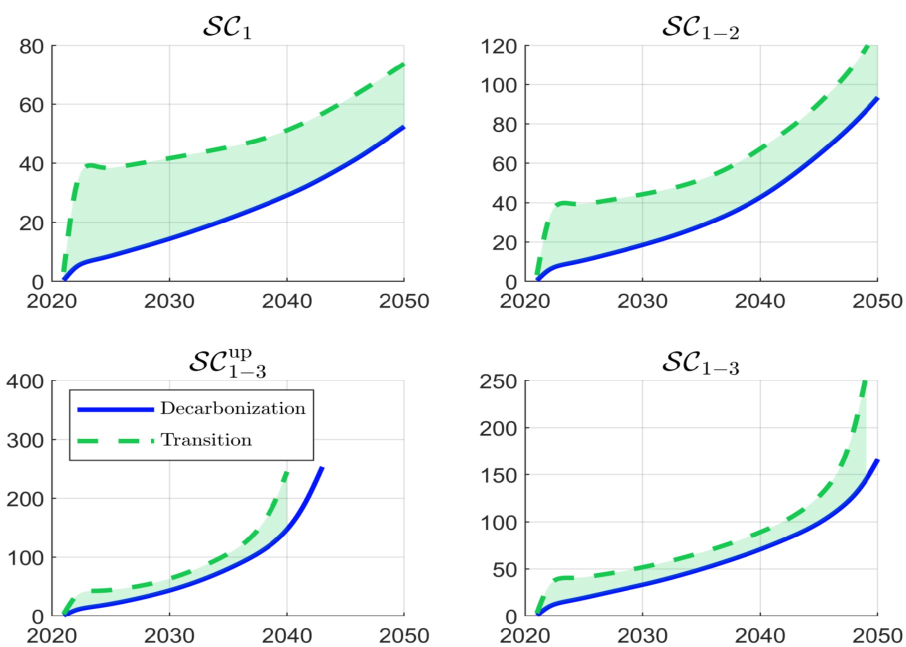


Figure 47: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3(0,10,2)$ constraint, $\mathcal{G} = 200\%$, $\mathcal{CM}^{\star} = -7\%$, CTB)

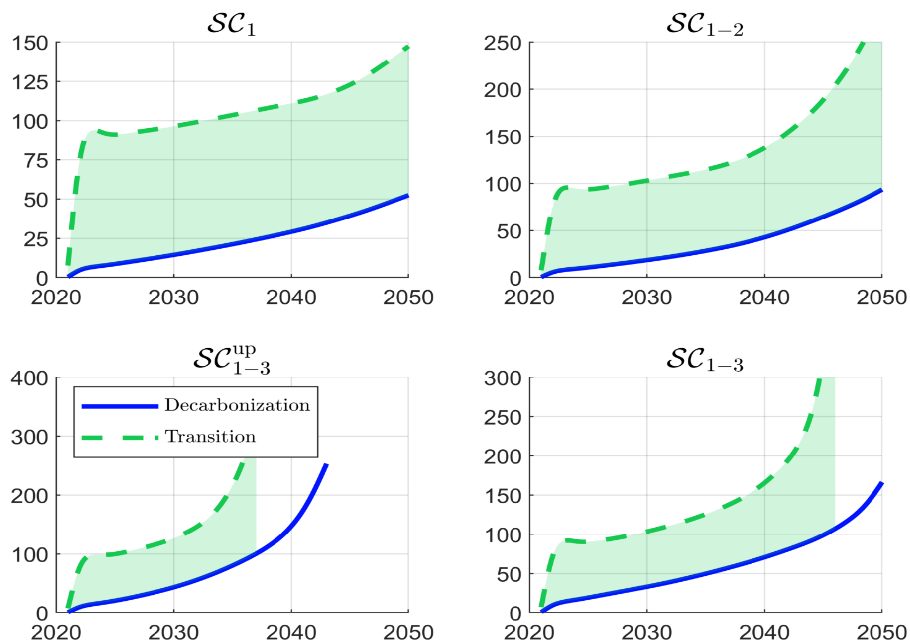


Figure 48: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)

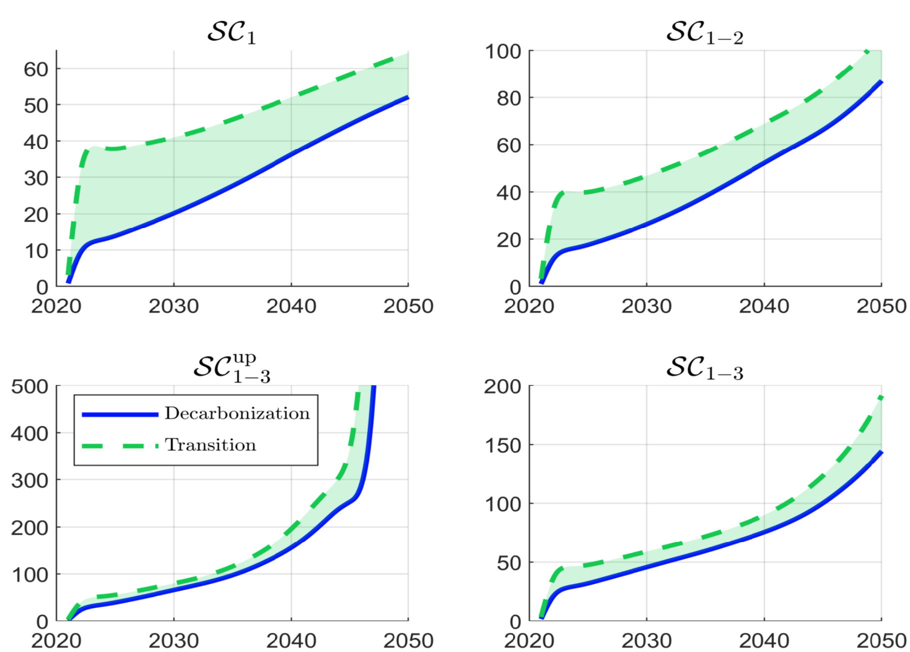


Figure 49: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 200\%$, $\mathcal{CM}^{\star} = -7\%$, PAB)

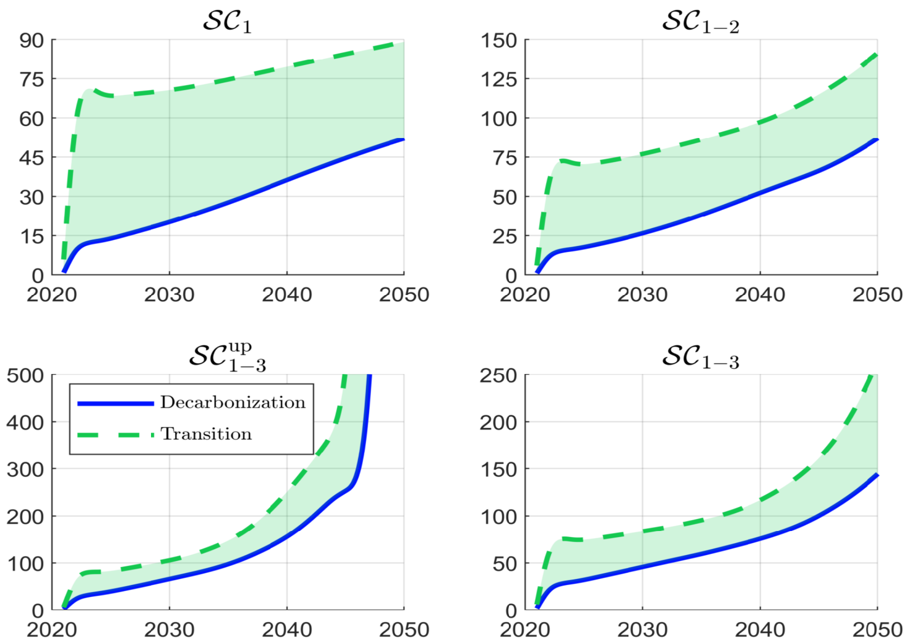


Figure 50: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)

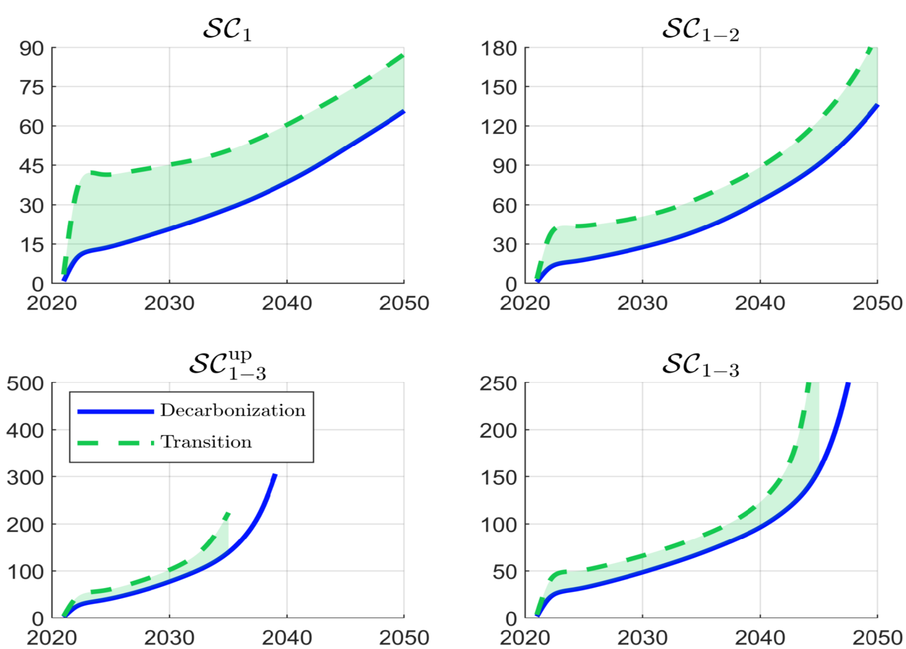


Figure 51: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 200\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -7\%$, PAB)

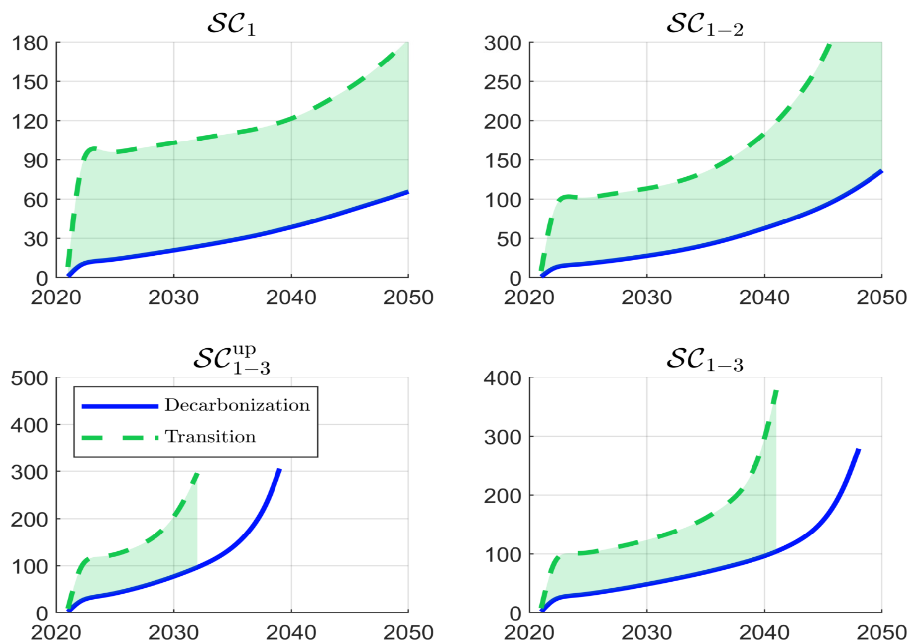


Figure 52: Tracking error volatility of net zero portfolios (MSCI EMU, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB)

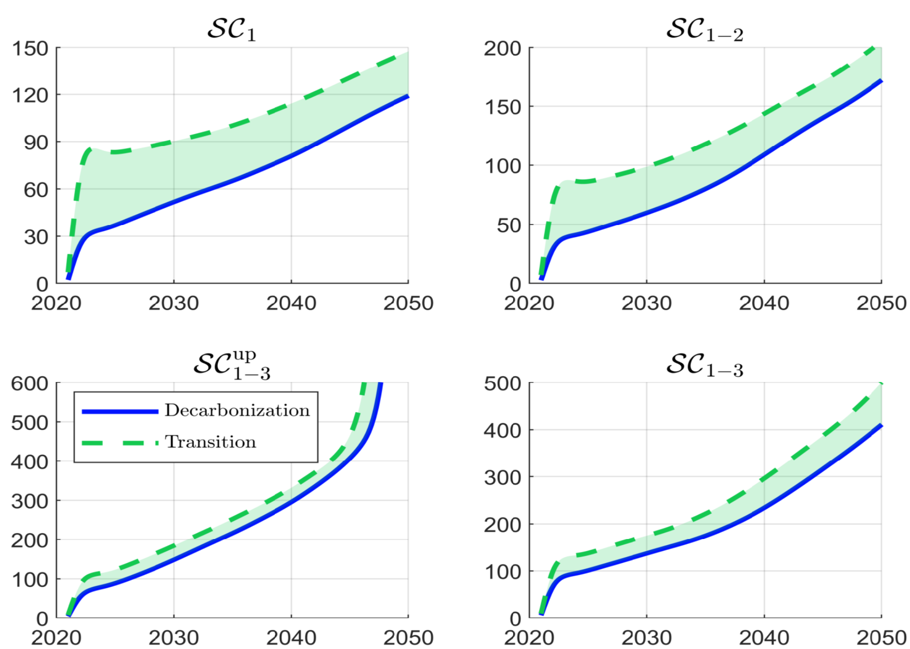


Figure 53: Tracking error volatility of net zero portfolios (MSCI EMU, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)

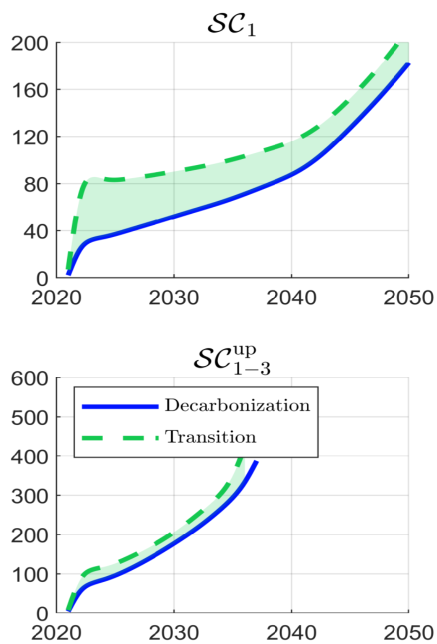


Figure 54: Tracking error volatility of net zero portfolios (MSCI USA, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)

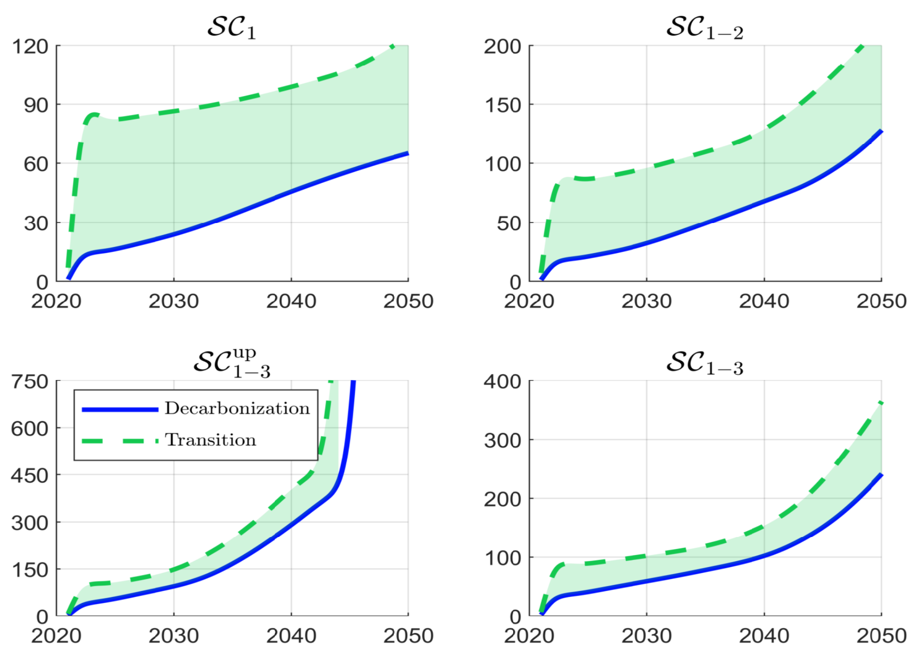
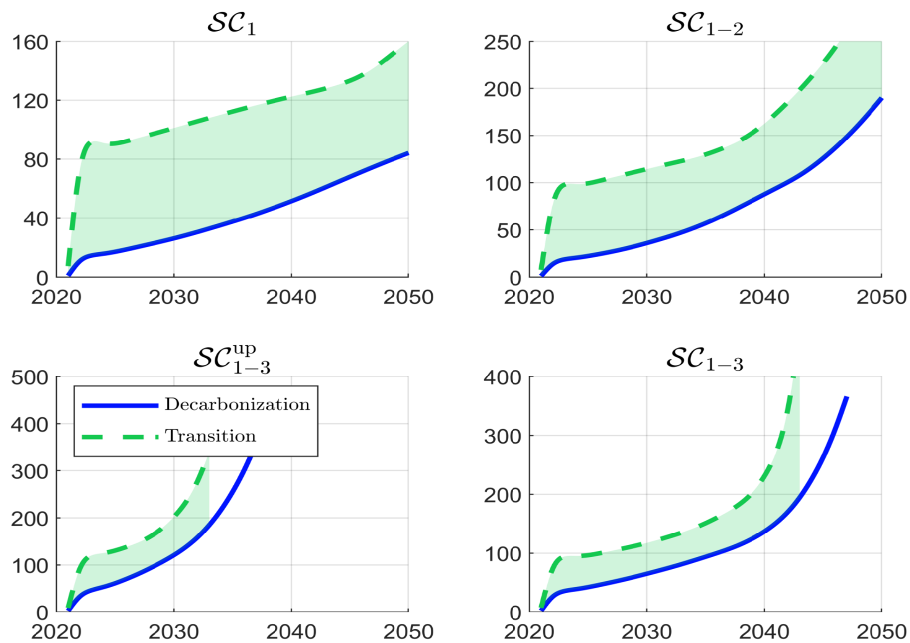


Figure 55: Tracking error volatility of net zero portfolios (MSCI USA, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB)



B.2.5 Integrated approach shrinkage

Figure 56: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_1$)

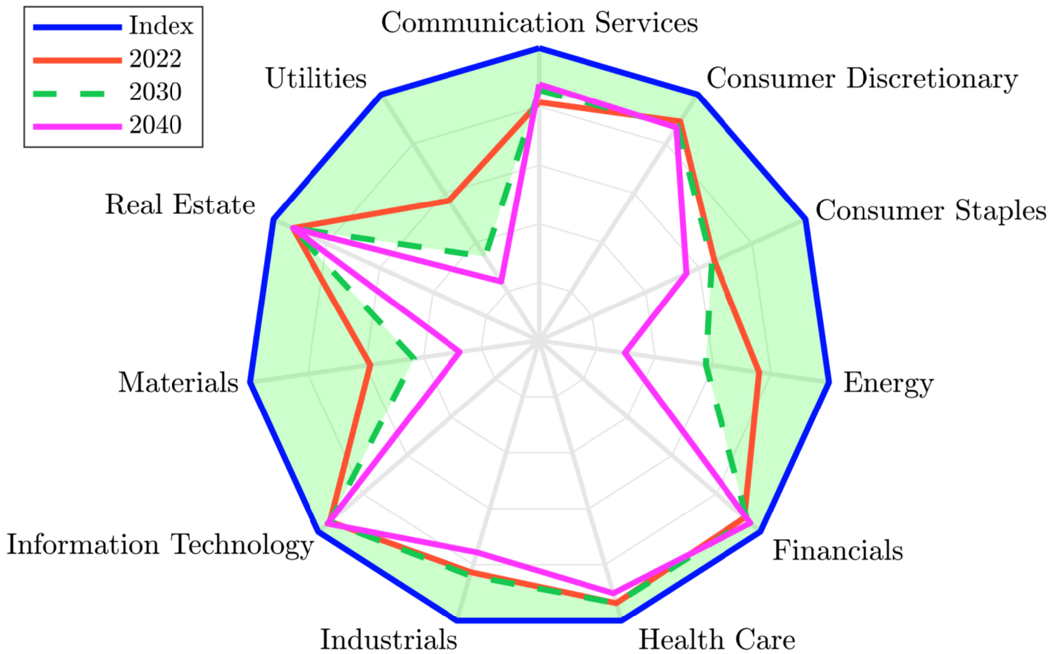


Figure 57: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-2}$)

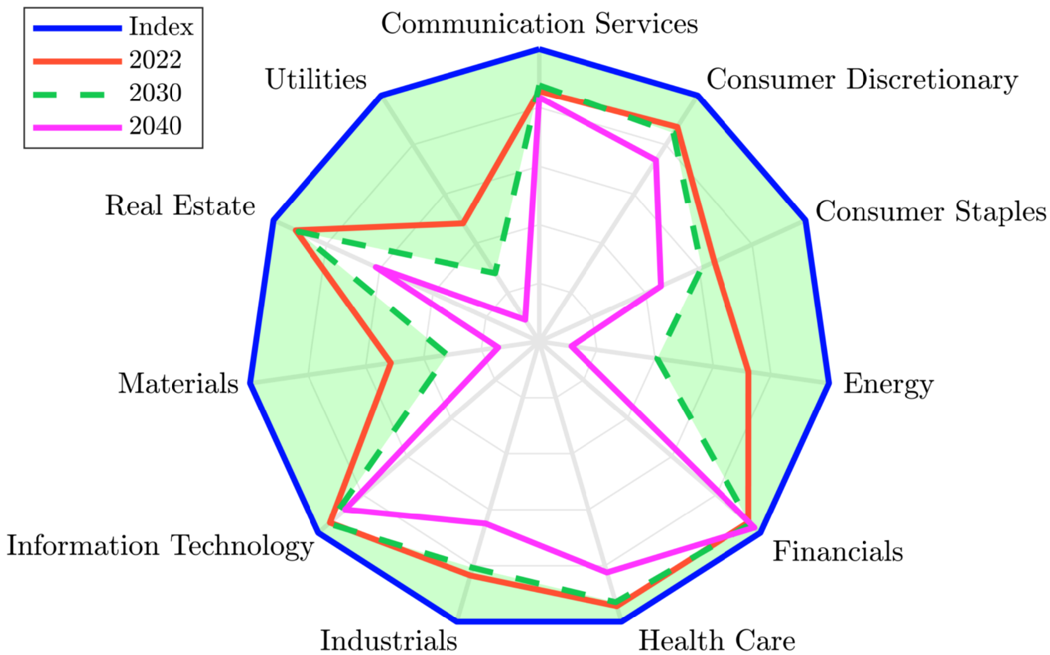


Figure 58: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}^{\mathrm{up}}$)

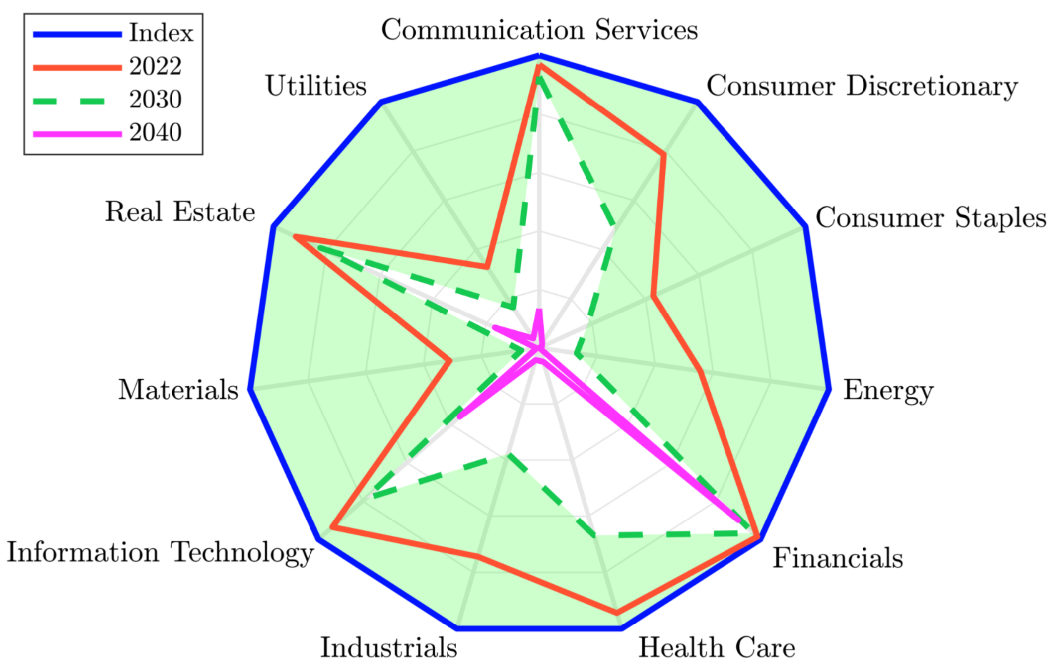


Figure 59: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

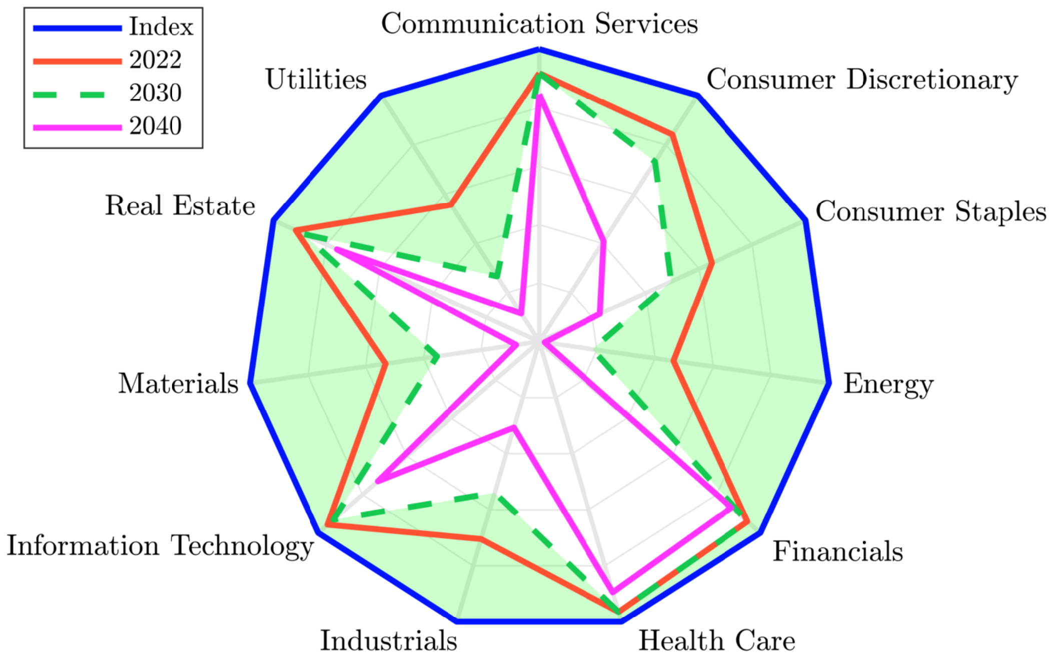


Figure 60: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_1$)

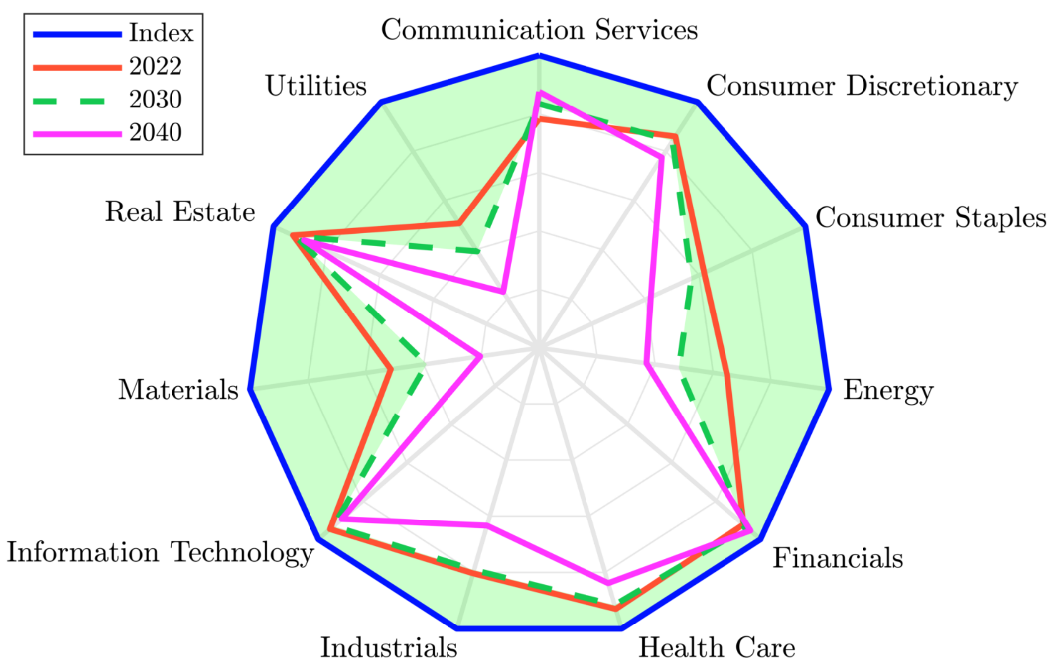


Figure 61: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-2}$)

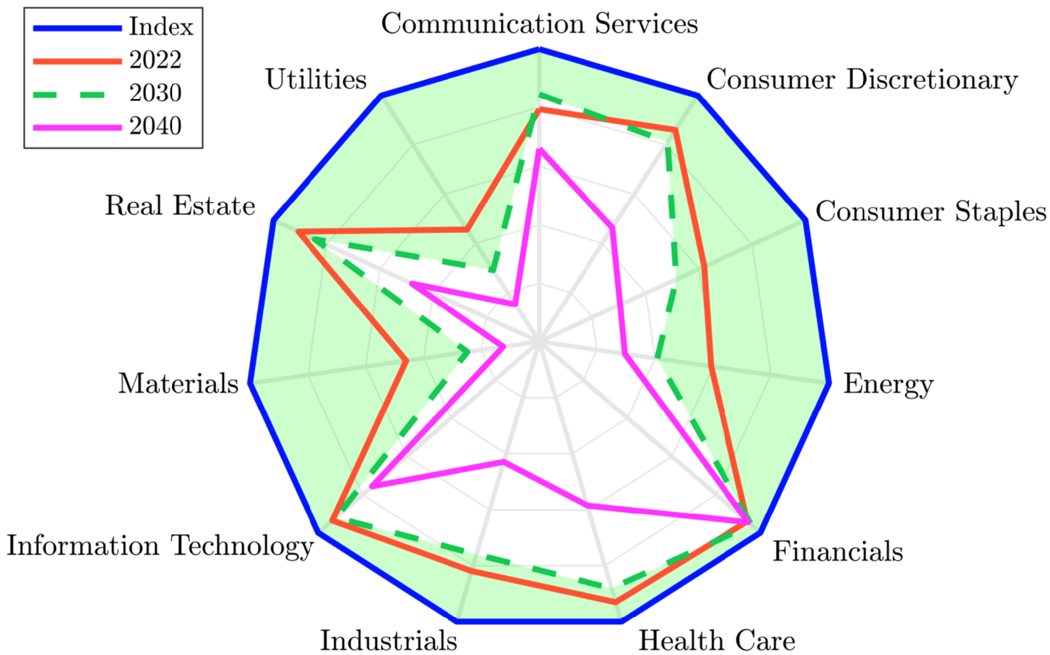


Figure 62: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}^{\text{up}}_{1-3}$)

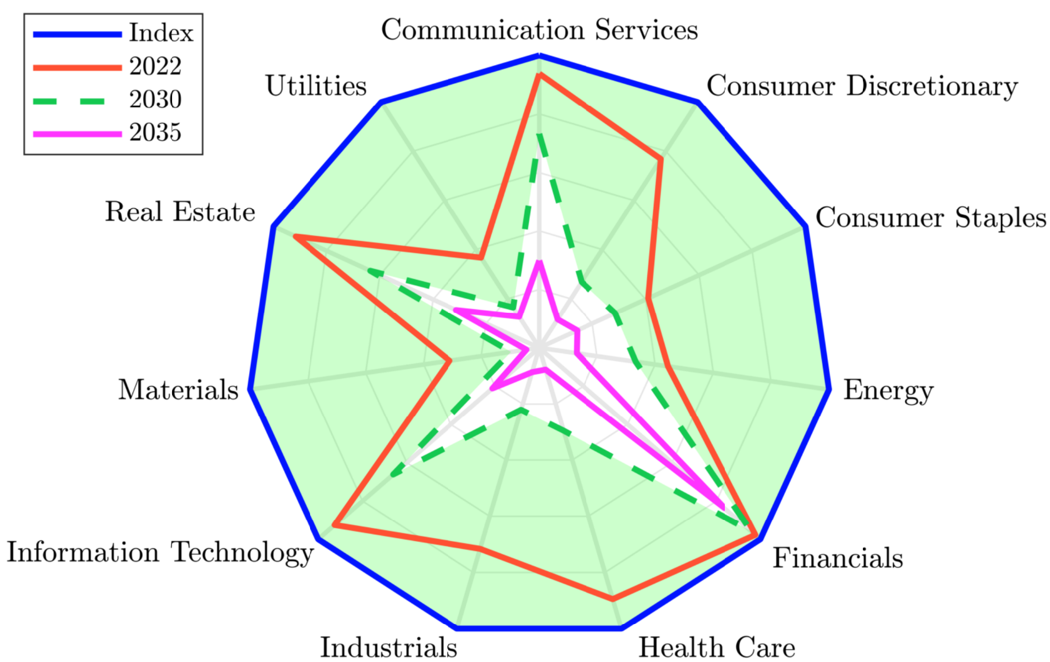


Figure 63: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

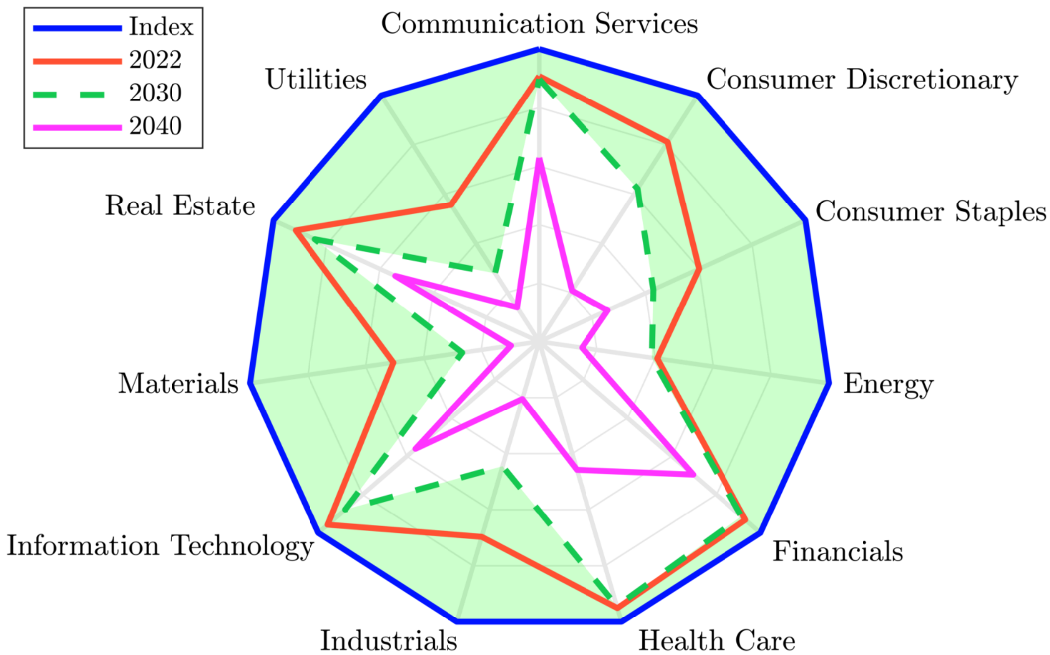


Figure 64: Radar chart representation of investment universe shrinkage (MSCI EMU, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

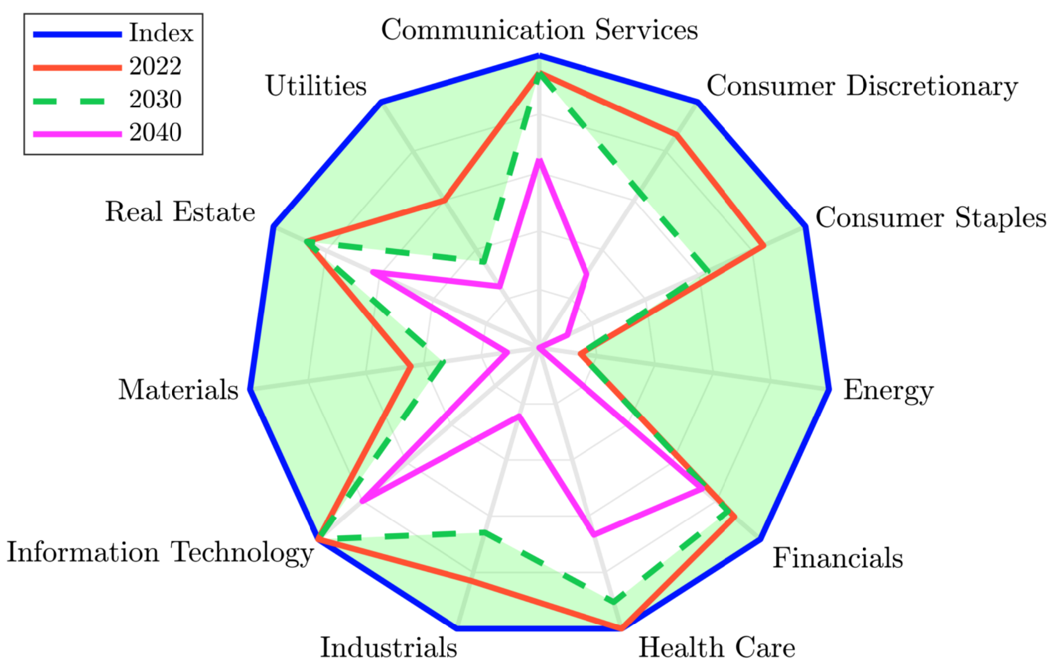


Figure 65: Radar chart representation of investment universe shrinkage (MSCI EMU, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

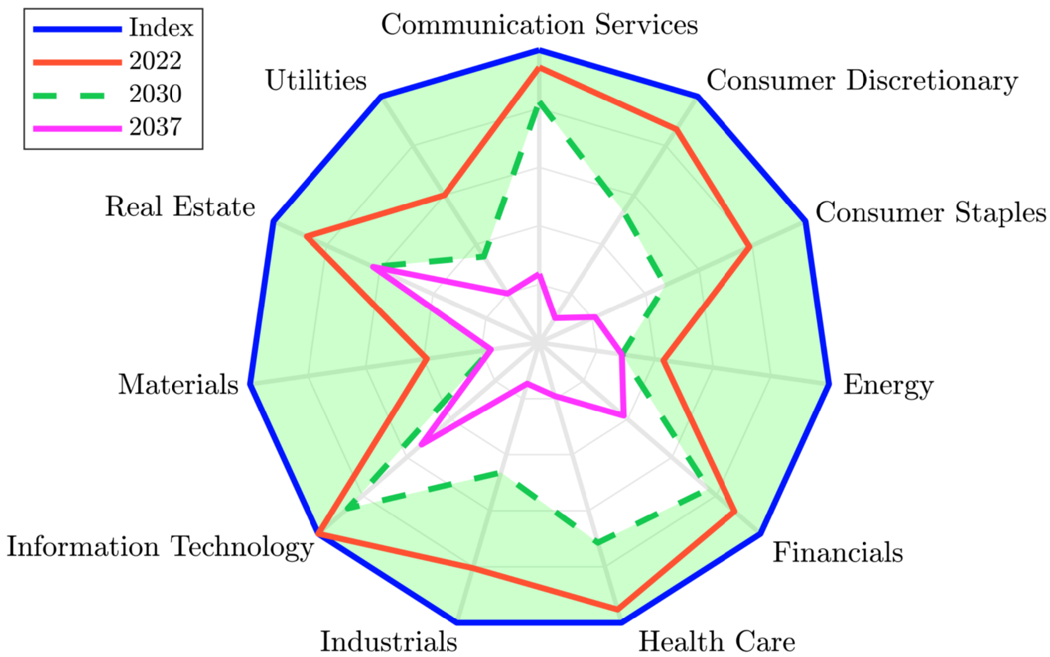


Figure 66: Impact of the momentum exclusion constraint on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

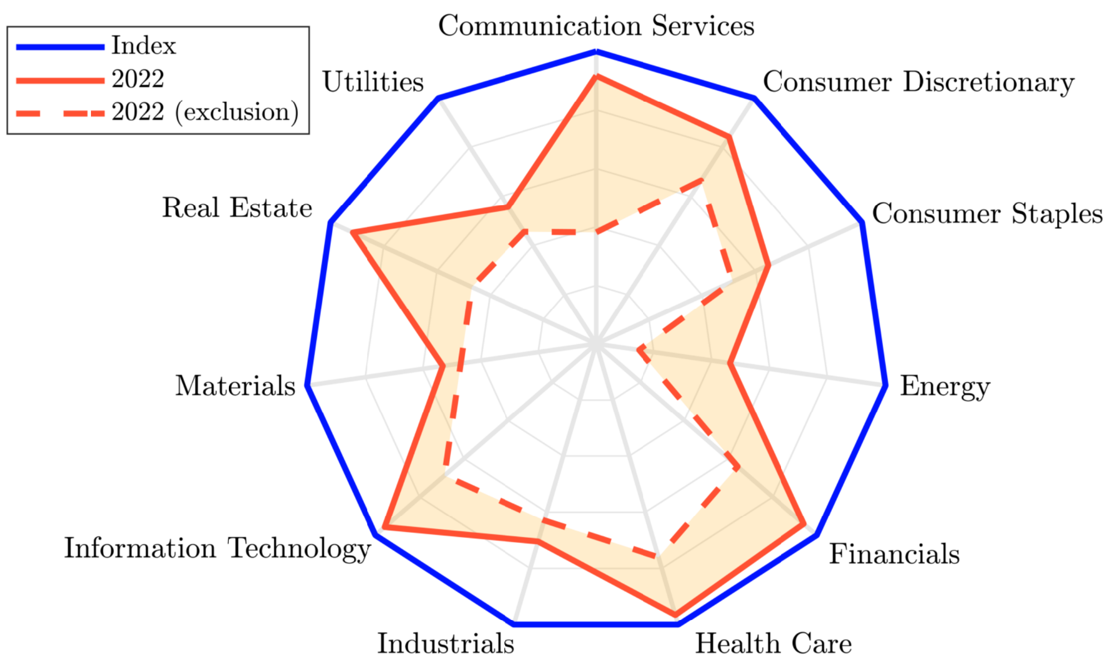


Figure 67: Impact of the momentum exclusion constraint on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)

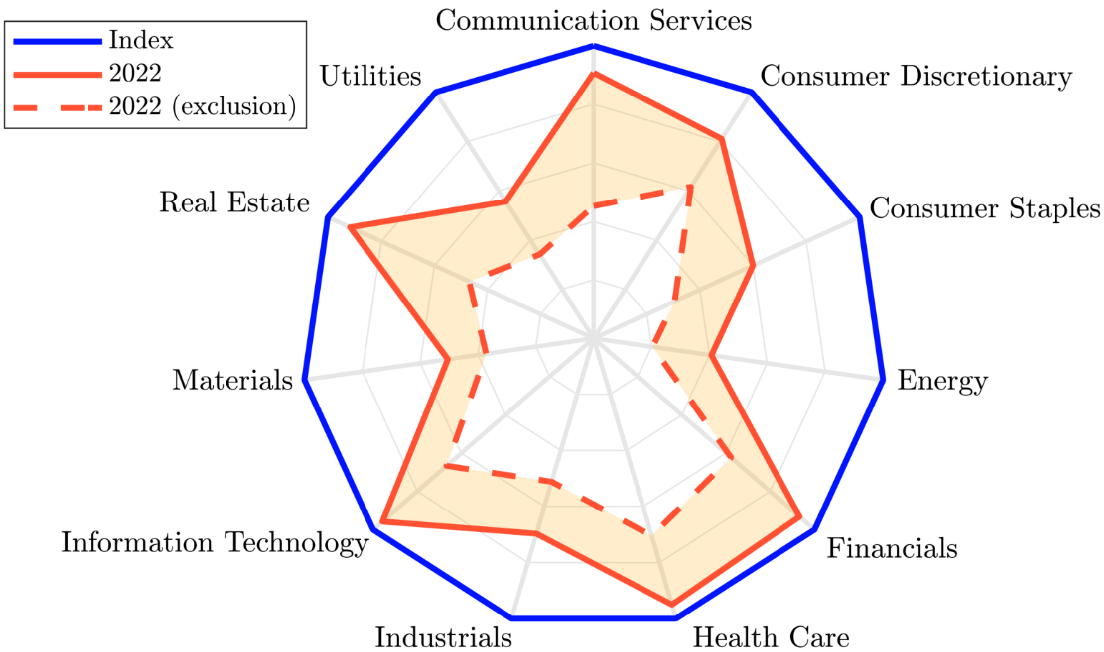


Figure 68: Case #1: $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$ vs. Case #2: $\mathcal{G} = 200\%$, $\mathcal{CM}^\star = -7\%$ (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB, scope $\mathcal{SC}_{1-3}$)

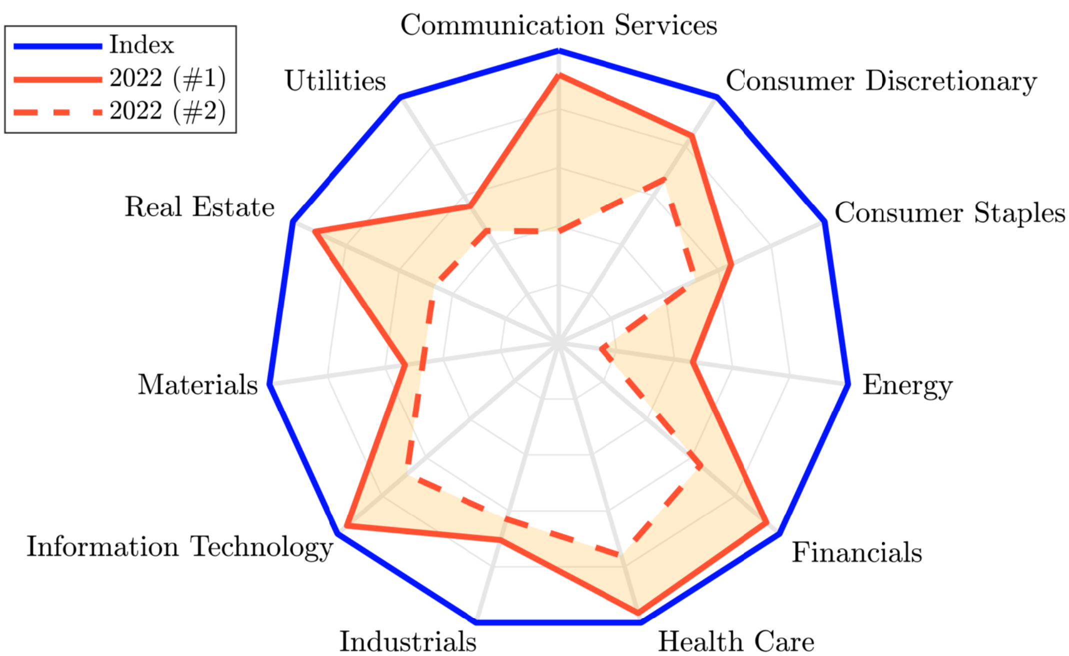


Figure 69: Case #1: $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$ vs. Case #2: $\mathcal{G} = 200\%$, $\mathcal{CM}^{\star} = -7\%$ (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB, scope $\mathcal{SC}_{1-3}$)

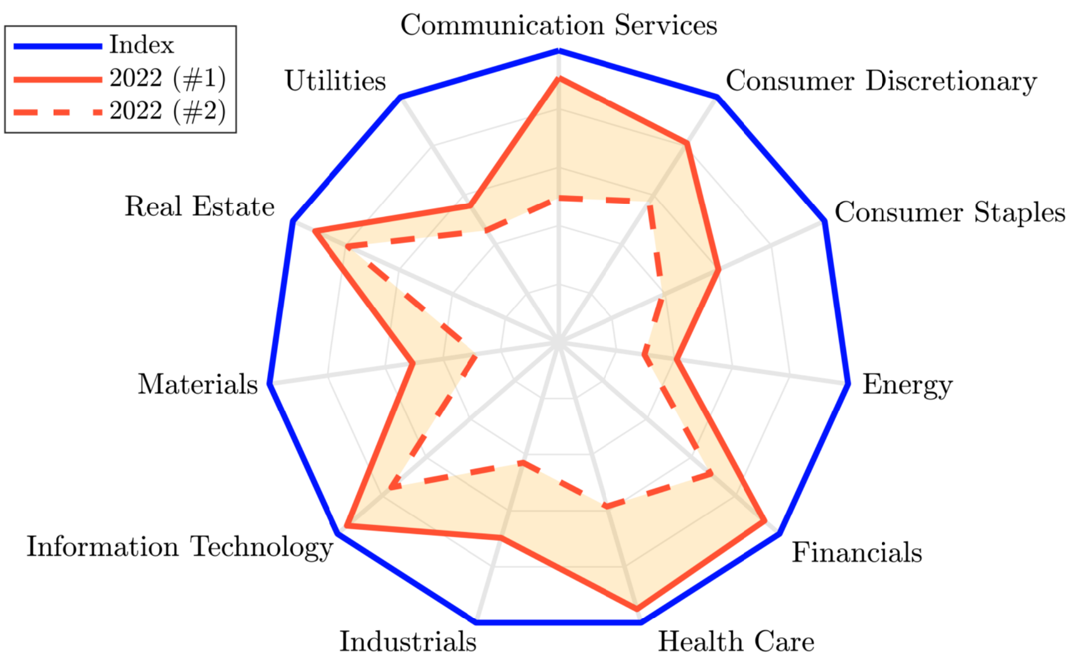


Figure 70: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)

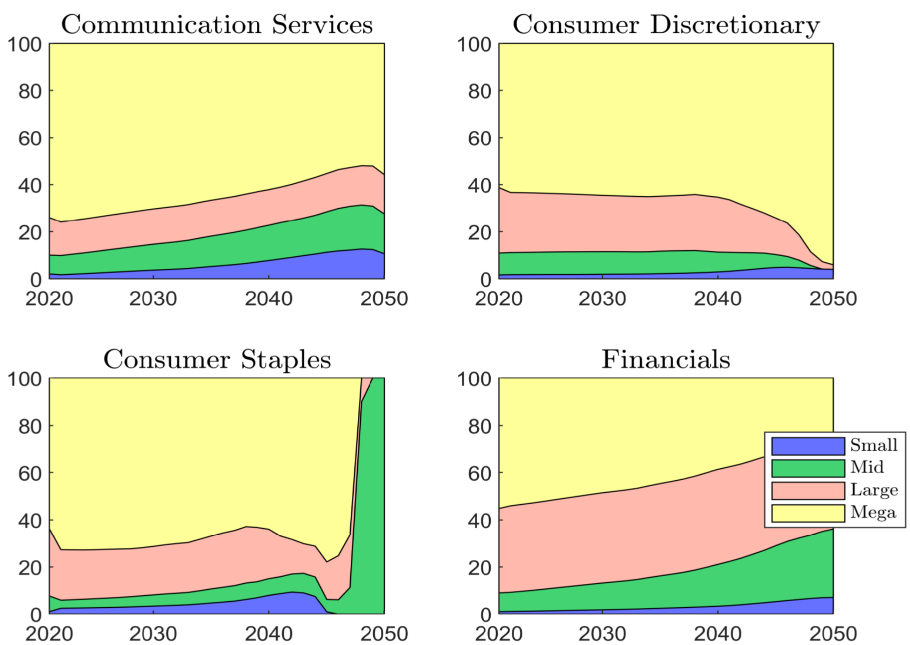


Figure 71: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

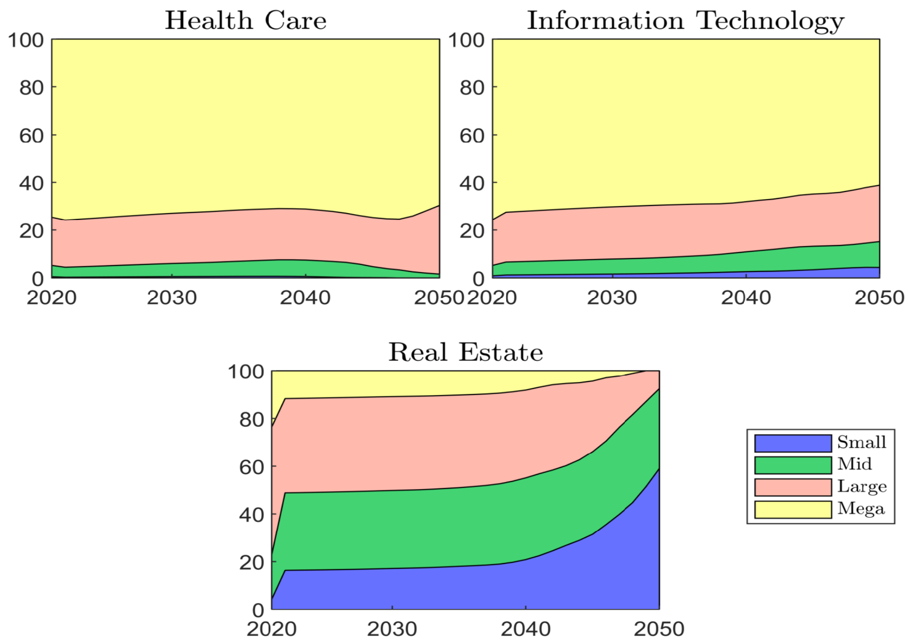


Figure 72: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

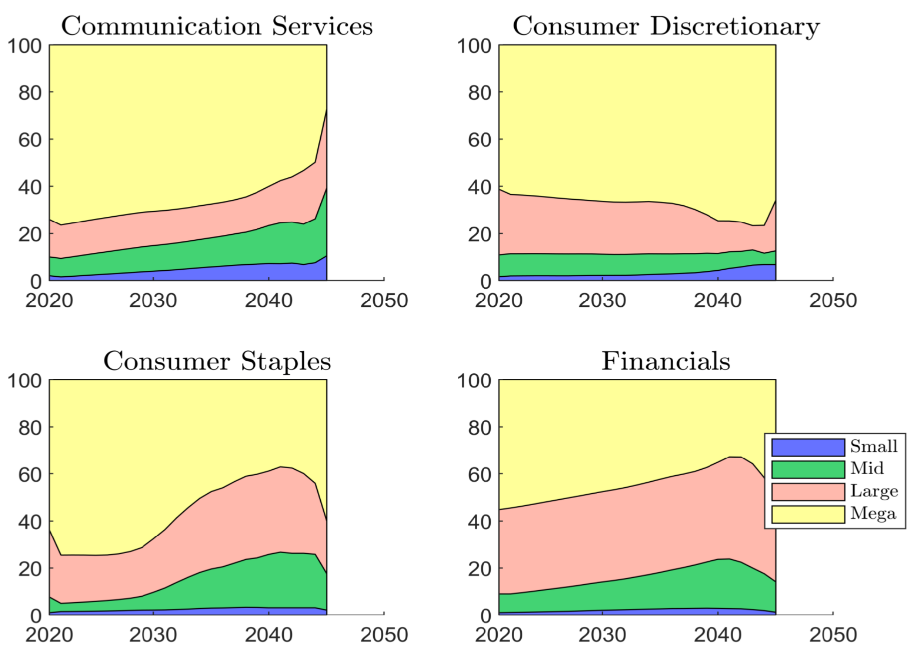
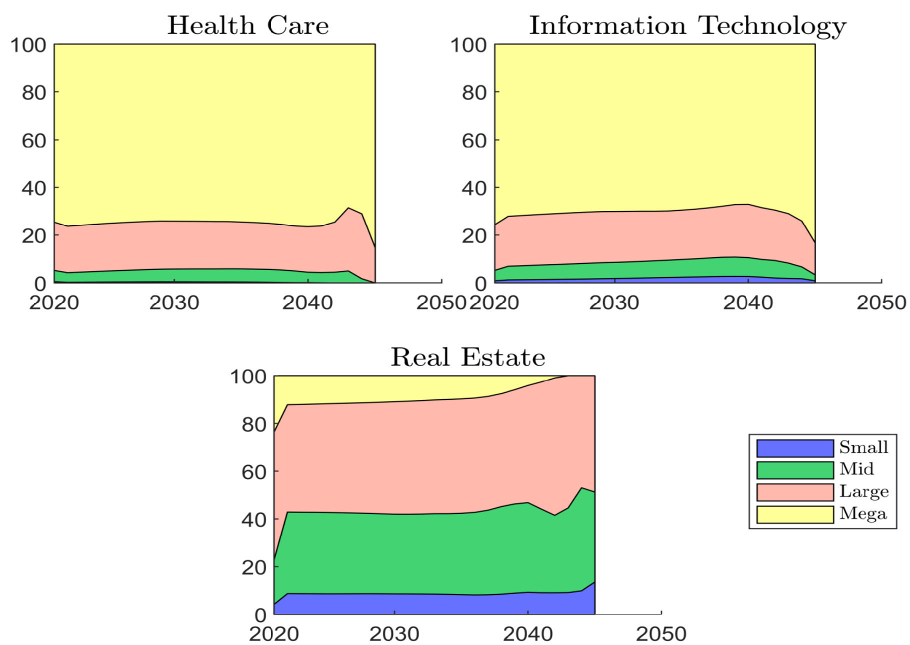


Figure 73: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_3\left(0,10,2\right)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



C Note de synthèse

Le risque climatique

Les enjeux face au réchauffement climatique sont cruciaux et le secteur financier a un rôle décisif à jouer dans le financement de la transition énergétique vers une économie décarbonée. En témoigne le développement du Network of Central Banks and Supervisors for Greening the financial System (NGFS), un groupe de banques centrales et de superviseurs volontaires qui souhaitent partager de bonnes pratiques et contribuer au développement de la gestion des risques financiers liés au changement climatique.

Les régulateurs peuvent en effet intervenir de deux manières : en contribuant à la création d'un environnement propice à la transition écologique, en particulier en surveillant les engagements des institutions financières et en garantissant la transparence de leur exposition, ainsi qu'en garantissant la protection des institutions financières contre les risques climatiques pour préserver la stabilité financière en vérifiant que les institutions ont identifié ces risques et mis en place des méthodes dédiées à leur gestion. Ce dernier point a récemment été largement étudié, notamment à travers un exercice de stress-test climatique mené par l'Autorité de Contrôle Prudentiel et de Résolution (ACPR) en 2020.

L'objectif principal de cet exercice est de sensibiliser les banques et les compagnies d'assurance aux risques auxquels elles devront faire face avec le changement climatique afin qu'elles intègrent cet aspect à long terme dans leur gouvernance et leur stratégie. L'objectif est de mettre en évidence la vulnérabilité des institutions aux différents scénarios climatiques. Ces scénarios suggèrent des transitions plus ou moins brutales pour lutter contre les émissions de gaz à effet de serre, en prenant comme référence la trajectoire tracée par les Accords de Paris visant à limiter le réchauffement mondial à moins de 1.5°C.

En étudiant plusieurs scénarios, le stress-test permet de mesurer le coût d'une trajectoire qui s'écarte des politiques définies par les accords de la COP21. Avant d'examiner ces scénarios, l'exercice pilote décline les risques climatiques en deux catégories : le risque de transition, lié aux changements de comportement des agents économiques et financiers nécessaires pour réduire les émissions de gaz à effet de serre, et le risque physique, lié aux impacts directs du changement climatique sur les biens et personnes.

La reconnaissance du changement climatique comme un risque majeur du 21ème siècle a largement été accompagné par la mise en place d'un cadre réglementaire exigeant l'évaluation de ces risques par les acteurs sur les marchés financiers (MiFID 2) mais également au sein des compagnies d'assurance (Solvabilité 2, Directive sur la Distribution d'Assurance). Les études sur le sujet se concentrent aujourd'hui principalement sur les risques physiques que les assureurs peuvent rencontrer en raison des conditions météorologiques extrêmes liées au changement climatique. Cela s'explique par les compétences qu'ont développées ces acteurs au cours de la pratique de leurs activités traditionnelle, notamment leur expertise autour des catastrophes naturelles qui les conduit à régulièrement collaborer avec des experts de leur domaine (e.g. météorologues etc.).

Cependant, il y a eu moins d'études sur les risques liés à la transition vers une économie à faibles émissions carbone qui peuvent affecter les gestionnaires d'actifs ainsi que les portefeuilles de marché des assureurs.

Net zéro

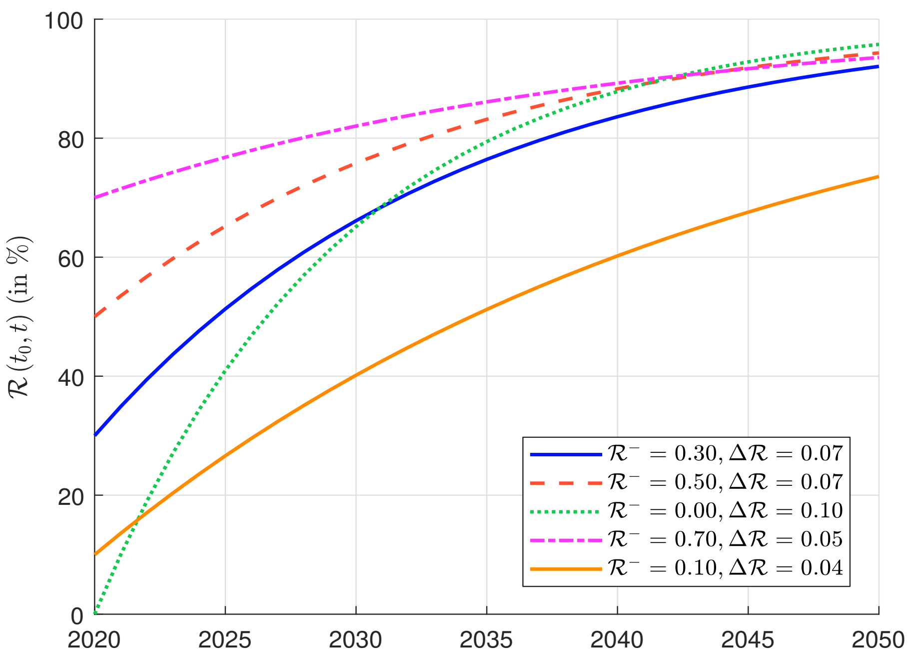
Les institutions financières se concentrent aujourd'hui sur des objectifs dit "net zéro" (pour zéro émission carbone) d'ici 2050, conformément à la limitation du réchauffement climatique à 1.5°C au cours de ce siècle. Toutefois, les récentes études sur le sujet ont mis en exergue diverses approches tant dans la définition de ces objectifs que les moyens mis en place pour les atteindre. Cette absence de définitions et de mesures communes, ainsi que de preuves de l'efficacité des stratégies, constitue un problème. En l'absence d'une compréhension commune et devant le vaste paysage actuel des mesures net zéro, il est difficile pour les parties prenantes de comparer leurs objectifs et d'évaluer si les mesures entreprises sont suffisantes pour parvenir à une économie mondiale neutre d'ici 2050. Par ailleurs, il semble irréaliste d'envisager atteindre la neutralité carbone à travers une réduction brutale des émissions uniquement, sans financer la transition vers une économie durable. C'est en cela que les objectifs d'une politique net zéro dépasse la simple décarbonation du système. Nous devons donc définir une telle politque en prenant compte de ses deux dimensions : la décarbonation du système et le financement de sa transition.

Ce mémoire d'actuariat vise alors à proposer une implémentation pratique d'une politique net zéro pour les gestionnaires d'actifs. Dans un premier temps, nous devons définir le scénario net zéro à suivre. Ce scénario se traduit généralement par une trajectoire de décarbonation à horizon 2050. La réduction des émissions carbones est ainsi rythmée par une réduction initiale $\boldsymbol{\mathcal{R}}^-$ ainsi qu'un rythme de réduction annuelle $\Delta\boldsymbol{\mathcal{R}}$ selon la formule suivante :

$$\boldsymbol{\mathcal{R}}\left(t_0, t\right) = 1 - \left(1 - \Delta\boldsymbol{\mathcal{R}}\right)^{t-t_0}\left(1 - \boldsymbol{\mathcal{R}}^-\right)$$

ou t_0 est l'année de référence, t l'année cible, et $\boldsymbol{\mathcal{R}}\left(t_0, t\right)$ le taux de réduction entre t_0 et t.

Figure 74: Exemples de trajectoire de décarbonation (en %)



Les scénarios net zéro peuvent ainsi différer dans le choix des paramètres de réduction mais également dans la définition des émissions carbones à réduire. Une entreprise peut en effet choisir de réduire ses émissions en absolues ou relativement à une variable de normalisation. Cette mesure des émissions, dite en intensité carbone, se définit par un ratio entre les émissions absolues $\boldsymbol{\mathcal{CE}}\left(t\right)$ et la variable de normalisation $Y\left(t\right)$:

$$\boldsymbol{\mathcal{CI}}\left(t\right) = \frac{\boldsymbol{\mathcal{CE}}\left(t\right)}{Y\left(t\right)}$$

La réduction en intensité permet ainsi de mesurer les émissions et efforts d'une entreprise relativement à sa taille (capitalisation boursière, chiffre d'affaire etc.) et offre la possibilité de comparer des entreprises de tailles ou secteurs différents. Par ailleurs, les émissions absolues se déclinent en différentes catégories. Ces "*scopes*" vont des émissions directes de gaz à effets de serres provenant de sources détenues et contrôlées par l'émetteur (scope 1) aux émissions indirectes suite à la distribution des produits de l'émetteur à travers leurs consommation par les clients et durées de vie (scope 3 *downstream* ou "aval").

Les deux scénarios principalement implémentés sur la place financière sont les labels Climate Transition Benchmark (CTB) et Paris Aligned Benchmark (PAB). Ces derniers correspondent tous les deux à un rythme de réduction annuelle de l'intensité carbone de 7% mais divergent par leur réduction initiale respective de 30% et 50%. De nombreuses discussions quant au scope d'émissions à prendre en compte dans ces scénarios net zéro subsistent et nous expliquons cela par le manque de fiabilité et transparence des données disponibles. Ce mémoire illustre notamment la difficulté d'inclure les émissions de toute la chaîne de valeur de l'émetteur (scope 3 aval) en montrant l'instabilité des données selon les fournisseurs. Une autre critique dirigée vers ces scénarios de décarbonation en intensité est leur manque de fondement physique car les scénarios de neutralité carbone de l'Agence Internationale de l'Énergie (AIE) sont en réalité construits en terme de budget carbone. Ces derniers définissent ainsi une limite de gaz à effet de serre en $GtCO_2e$[56] à respecter d'ici 2050. Si à première vue, la neutralité carbone en intensité équivaut à la neutralité en émissions, nous montrons que, sous nos hypothèses, les trajectoires PAB et CTB sont plus agressives que les scénarios de l'AIE.

D'autre part, ce mémoire explore différentes pistes pour diriger ses investissement vers des entreprises qui participent à la transition vers une économie bas carbone. Contrairement à l'estimation des émissions, la mesure de la capacité à transitionner d'un émetteur échoue à se généraliser. En effet, une entreprise peut parfois difficilement être un acteur de la transition tout en étant clé pour l'économie, de part son secteur d'activité notamment, comme le secteur de la santé. Cela illustre le besoin mais également la difficulté de définir des mesures simples de "l'intensité verte" d'un émetteur. Dans le cadre de ce mémoire, nous présentons et nous reportons à la taxonomie verte européenne construite par le Groupe d'Experts Européen (TEG), aujourd'hui toujours en développement . L'objectif de cette dernière est alors dans un premier temps de définir un ensemble d'activités définies comme vertes et permettre aux entreprises de faire remonter un revenu vert $\boldsymbol{\mathcal{GRS}}$ issue de ces activités. A partir de cette métrique, nous définissons l'intensité verte d'un émetteur comme la part de son revenu issue de ces activités :

$$\boldsymbol{\mathcal{GRS}} = \frac{\boldsymbol{\mathcal{GR}}}{\boldsymbol{\mathcal{TR}}}$$

Enfin, pour compléter cette métrique et mesurer l'effort et la capacité d'un émetteur à réduire ses émissions, nous régressons son historique d'émissions sur le temps :

$$\boldsymbol{\mathcal{CE}}\left(t\right) = \beta_0 + \beta_1 \cdot t + u\left(t\right)$$

et en déduisons un coefficient de décarbonation idiosyncratique β_1. Nous normalisons ce coefficient par rapport aux dernières émissions absolues de l'émetteur par souci de comparaison entre différentes entreprises :

$$\boldsymbol{\mathcal{CM}}^{\mathcal{L}ong}\left(t\right) = \frac{\hat{\beta}_1\left(t\right)}{\boldsymbol{\mathcal{CE}}\left(t\right)}$$

Le rôle de ce coefficient est alors d'identifier les participants les plus à même de faire des efforts à partir de l'évolution de leurs émissions passées et de les inclure dans nos portefeuilles. Il s'agit ainsi non seulement de ne pas exclure les émetteurs importants qui réduisent fortement leur émissions mais aussi de construire un portefeuille avec un niveau d'auto-décarbonation satisfaisant. Nous pourrions ainsi, sans énormément modifier nos positions, nous assurer de financer un portefeuilles d'entreprises sur le chemin de la transition au rythme souhaité.

[56]équivalent méga tonne de dioxyde de carbone

Méthodologie

La littérature inscrit aujourd'hui le risque climatique comme un facteur de risque ordinaire venant s'imbriquer dans les outils traditionnels des institutions financières. Il s'agit alors dorénavant de mesurer ce risque dans les portefeuilles *business-as-usual* afin d'étudier leur capacité à intégrer le risque climatique et d'y apporter les modifications nécessaires à leur alignement à des politiques net zéro. Cette approche traduit finalement la volonté des acteurs financiers à modifier à la marge leurs portefeuilles afin de les faire rentrer dans les nouvelles contraintes réglementaires plutôt que de considérer le risque climatique ou la neutralité carbone comme un cadre d'investissement à part entière.

Afin de traduire cette volonté de proximité entre un portefeuille aligné et le *business-as-usual*, ce mémoire emprunte la méthodologie de construction de portefeuilles de Markowitz en présence d'un portefeuille de référence (*benchmark*). L'objectif est alors de construire un portefeuille proche du portefeuille de référence, qui représente ici un exemple de portefeuille géré par une institution financière, mais respectueux de diverse contraintes climatiques définies à l'aide des métriques mises en avant dans ce mémoire (émissions en intensité, part de revenus verts, auto-décarbonation etc.).

Considérons un univers de n actifs investissables. Soit $b = (b_1, \ldots, b_n)$ le vecteur des poids de chaque actif dans le portefeuille. Dans le cadre de ce mémoire, le portefeuille de référence b investit dans chaque actif en proportion de leur capitalisation boursière. Nous notons $R = (R_1, \ldots, R_n)$ le vecteur de rendements des actifs de l'univers où R_i est le rendement de l'actif i. Le rendement espéré du portefeuille de référence est alors :

$$\mu(b) = \mathbb{E}\left[b^\top R\right] = b^\top \mu$$

Avec $\mu = \mathbb{E}[R]$, tandis que sa variance vaut :

$$\sigma^2(b) = b^\top \Sigma\, b$$

avec $\Sigma = \mathbb{E}\left[(R-\mu)(R-\mu)^\top\right]$ la matrice de covariance des rendements des actifs.

Nous cherchons maintenant un portefeuille x proche du portefeuille de référence b. Une mesure traditionnelle de la proximité de ces deux portefeuille est l'erreur de réplication, ou *tracking error*. Si l'on note $\mu\left(x \mid b\right) = (x-b)^\top \mu$ l'excès de rendement du portefeuille x, nous pouvons définir la volatilité de l'erreur de réplication du portefeuille x par rapport à b par :

$$\sigma\left(x \mid b\right) = \sqrt{(x-b)^\top \Sigma\,(x-b)}$$

L'objectif est alors de minimiser cette erreur.

Construction d'un portefeuille actions

Nous combinons à ce problème de minimisation diverses contraintes de construction de portefeuille ainsi que les contraintes climatiques de décarbonation et de transition[57]. Le problème s'écrit alors:

$$\begin{aligned} x^\star(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^\top \Sigma(t)\left(x - b(t)\right) \qquad (93)\\ &\text{s.t.} \left\{ \begin{array}{ll} \boldsymbol{\mathcal{CI}}(t,x) \le \left(1 - \boldsymbol{\mathcal{R}}(t_0,t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{ Décarbonation} \\ x \in \Omega_{\boldsymbol{\mathcal{T}}ransition}(t) & \longleftarrow \text{ Transition} \\ x \in \Omega_1 \cap \Omega_2(t) & \end{array} \right. \end{aligned}$$

[57]cette méthodologie est alors adaptée en définissant de nouvelles métriques dans le cas d'un portefeuille obligataire.

où la décarbonation suit les trajectoires imposées par les labels PAB ou CTB et la transition est définie par l'ensemble $\Omega_{\mathcal{T}ransition}(t)$. Nous définissons notamment cet ensemble par :

$$x \in \Omega_{\mathcal{T}ransition}(t) \Leftrightarrow \left\{ \begin{array}{ll} \mathcal{GI}(t,x) \geq \left(1+\mathcal{G}(t)\right) \cdot \mathcal{GI}\left(t_0, b\left(t_0\right)\right) & \longleftarrow \quad \text{Greenness} \\ \mathcal{CM}^{\mathcal{L}ong}(t,x) \leq \mathcal{CM}^{\star} & \longleftarrow \quad \text{Momentum} \end{array} \right. \tag{94}$$

Enfin, l'ensemble $\Omega_1 \cap \Omega_2(t)$ définit des contraintes d'optimisation traditionnelles de construction de portefeuilles (portefeuille *long only*, limite de déviation sectorielle, limite de déviation par rapport au portefeuille de référence etc.).

Conclusions

En empruntant une approche traditionnelle de construction de portefeuille qui vise à minimiser l'erreur de réplication, ce mémoire d'actuariat permet de tirer les enseignements suivants sur l'implémentation de portefeuilles net zéro.

Sensibilité à la méthode de calcul des émissions Le premier enseignement concerne la sensibilité de la solution aux paramètres et aux données. En particulier, les gestionnaires de fonds doivent être prudents lorsqu'ils choisissent le périmètre d'émissions carbone qu'ils utilisent pour décarboner leurs portfeuilles. Le net zéro n'a de sens que s'il concerne l'ensemble des émissions d'un système fermé. Par conséquent, ce sont les émissions scope 3 qui doivent être prises en compte pour aligner un portefeuille par rapport à un scénario de neutralité carbone. Le problème est que nous observons aujourd'hui un manque de fiabilité autour de ces données d'émissions. Néanmoins, il est important que les acteurs financiers (gestionnaires, régulateurs etc.) commencent à les utiliser afin de créer des incitations à en améliorer la qualité. Cependant, ce mémoire montre que l'inclusion des émissions scope 3 augmente le risque d'erreur de réplication, en particulier pour les émissions en amont (*upstream*). De même, le portefeuille résultant dépend fortement de la cible que nous souhaitons atteindre en terme de part de revenus verts et niveau d'auto-décarbonation. Les gestionnaires de fonds doivent alors être prudents car une trop grande ambition à court terme implique qu'il pourrait ne pas y avoir de solution à moyen terme au problème d'optimisation, et donc pas d'alignement possible.

La distinction entre décarbonation et alignement Le second résultat clé est que la décarbonation et l'alignement des portefeuilles donnent des solutions différentes car ce sont deux politiques d'investissement distinctes. En particulier, la décarbonation d'un portefeuille est plus facile que son alignement. Nous montrons notamment que la décarbonation selon les trajectoires CTB ou PAB ne conduit jamais à une explosion des *tracking errors* jusqu'en 2030. En fait, le véritable enjeu de l'exercice de décarbonation réside dans le risque de diversification et de liquidité auquel un investisseur peut être confronté. Ces risques sont amplifiés lorsque nous ajoutons la dimension de transition dans le programme d'optimisation. Au-delà d'un coût de *tracking error* plus élevée, il n'y a même aucune garantie qu'une solution existe toujours. En outre, l'introduction du pilier de la transition souligne la difficulté de choisir un ensemble approprié de contraintes pour les portefeuilles net zéro, car certains paramètres peuvent être négativement corrélés avec d'autres.

Nos résultats sont par ailleurs en ligne avec la littérature : la décarbonation des portefeuilles est systématiquement une stratégie longue sur les émetteurs financiers et courte sur les émetteurs d'énergie, de matériaux et de services publics. Par conséquent, nous nous retrouvons dans une situation dans laquelle la capacité de transition d'un portefeuille décarboné est plus faible que celle du portefeuille de référence, car les solutions vertes sont également situées dans des secteurs à forte intensité carbone. Il est donc crucial de faire la distinction entre les émetteurs à forte empreinte carbone qui ne participeront pas à la transition et ceux qui réduiront leurs émissions et trouveront des solutions durables. Par ailleurs, comme la transition comporte de multiples facettes, les professionnels sont tentés de multiplier les métriques permettant de mesurer la capacité d'un portefeuille à la soutenir. Ce n'est malheureusement pas une approche fiable car ces métriques peuvent être indépendantes à court terme (comme l'auto-décarbonation et la part des revenus verts d'une entreprise). Il est ainsi préférable de se concentrer sur un nombre restreint de contraintes de transition et de comprendre l'objectif de chacune d'entre elles. Un problème concis pour définir le net zéro est plus utile qu'un cadre complexe et un empilement diffus de critères.

S'éloigner du *business-as-usual* Le troisième résultat principal est que la décarbonation et l'alignement du portefeuille sont deux processus d'exclusion. Cela signifie qu'il est tout à fait impossible d'atteindre un alignement net zéro sans permettre l'exclusion de certains actifs du portefeuille de référence. Par conséquent, certains acteurs clés de la transition tels que les entreprises du secteur de l'énergie et des services publics disparaissent subitement de l'univers investissable. C'est pourquoi, imposer une proximité à un portefeuille *benchmark* par la neutralité sectorielle peut entraîner problèmes à l'existence de solutions. Le processus d'exclusion que nous observons à la fois au niveau de l'émetteur et du secteur soulève alors une question d'étalonnage. En effet, si la décarbonation de portefeuille peut être considérée comme une légère inclinaison du portefeuille de référence, l'alignement du portefeuille peut elle impliquer un rétrécissement significatif de l'univers d'investissement. En tant que telle, ce rétrécissement montre que la définition de l'investissement net zéro est complexe car bien trop éloignée de l'investissement habituel. A court terme, l'écart reste important, et le choix de s'éloigner des méthodes *business-as-usual* est une question importante pour tous les investisseurs net zéro.

De nouvelles approches La dernière remarque concerne la mise en oeuvre de politiques d'investissement net zéro. Dans ce mémoire, nous nous sommes concentrés sur l'approche traditionnelle *top-down* pour sa simplicité à obtenir des résultats quantitatifs. Cependant, il ne s'agit pas de la seule approche envisageable. En particulier, la gestion active prend beaucoup de sens dans l'implémentation d'un investissement net zéro. Par exemple, nous avons présenté l'approche coeur-satellite, qui comprend la dimension de décarbonation pour l'investissement de base (coeur) et la dimension de transition pour la stratégie satellite. Ce cadre est plus facile à mettre en oeuvre que l'approche d'optimisation intégrée qu'explore ce mémoire. De plus, il permet de contrôler la répartition entre les deux dimensions, et de modifier progressivement le poids de la dimension de transition en fonction de l'écologisation de l'économie. Actuellement, le net zéro pourrait finalement être considéré comme un investissement thématique car l'univers des actifs de transition est encore petit. Mais à l'avenir, si une différence subsiste encore entre l'investissement net zéro et l'investissement traditionnel, cela signifierait que nous avons collectivement échoué à limiter le réchauffement climatique.

D Executive summary

Climate risk

The challenges of global warming are crucial and the financial sector has a decisive role to play in financing the energy transition to a low-carbon economy. This is evidenced by the development of the Network of Central Banks and Supervisors for Greening the Financial System (NGFS), a group of central banks and voluntary supervisors who wish to share best practices and contribute to the development of financial risk management related to climate change. Regulators can intervene in two ways: by contributing to the creation of an environment conducive to the ecological transition, in particular by monitoring the commitments of financial institutions and guaranteeing the transparency of their exposure, and by guaranteeing the protection of financial institutions against climate risks in order to preserve financial stability by verifying that institutions have identified these risks and put in place methods dedicated to their management. This last point has recently been extensively studied, notably through a climate stress-testing exercise conducted by the l'Autorité de Contrôle Prudentiel et de Résolution (ACPR) in 2020. The main objective of this exercise is to make banks and insurance companies aware of the risks they will have to face with climate change so that they integrate this long-term aspect into their governance and strategy. The objective is to highlight the vulnerability of institutions to different climate scenarios. These scenarios suggest more or less abrupt transitions to combat greenhouse gas emissions, taking as a reference the trajectory mapped out by the Paris Agreements aimed at limiting global warming to less than 1.5°C.

By examining several scenarios, the stress-test measures the cost of a trajectory that deviates from the policies defined by the COP21 agreements. Before examining these scenarios, the pilot exercise breaks down climate risks into two categories: transition risk, related to changes in the behavior of economic and financial agents needed to reduce greenhouse gas emissions, and physical risk, related to the direct impacts of climate change on goods and people. The recognition of climate change as a major risk of the 21st century has been largely accompanied by the implementation of a regulatory framework requiring the assessment of these risks by financial market players (MiFID 2) but also within insurance companies (Solvency 2, Insurance Distribution Directive). Studies on the subject today focus mainly on the physical risks that insurers may encounter due to extreme weather conditions linked to climate change. This is due to the skills that these actors have developed during the practice of their traditional activities, notably their expertise around natural catastrophes which leads them to regularly collaborate with experts in their field (e.g. meteorologists etc.). However, there have been fewer studies on the risks related to the transition to a low-carbon economy that may affect asset managers and insurers' market portfolios.

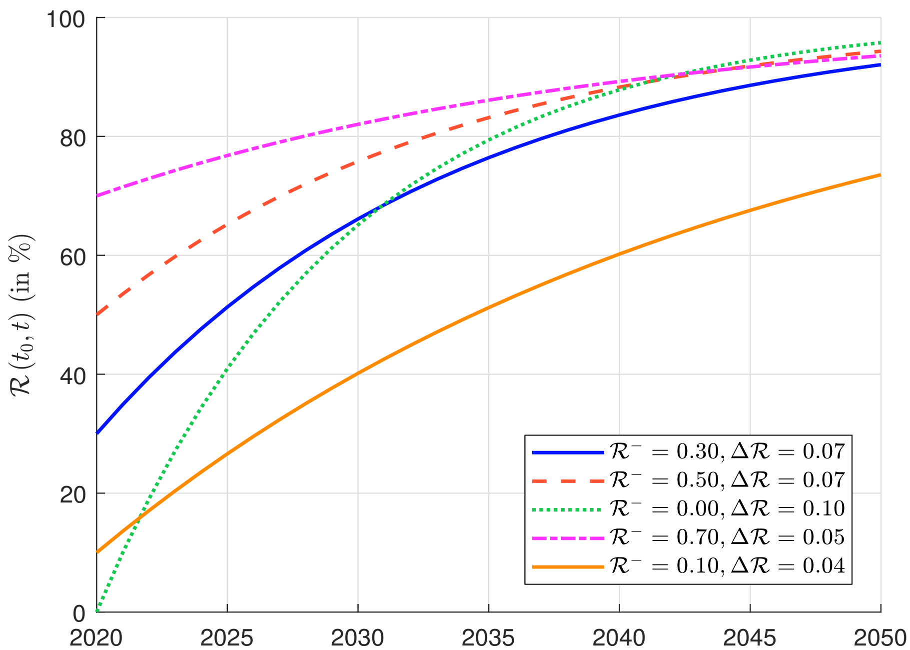
Net zero

Financial institutions are now focusing on so-called "net zero" (for zero carbon emissions) targets by 2050, consistent with limiting global warming to 1.5°C this century. However, recent studies on the subject have highlighted a variety of approaches to both the definition of these goals and the means of achieving them. This lack of common definitions and measures, as well as evidence of the effectiveness of strategies, is a problem. In the absence of a common understanding and the current broad landscape of net zero actions, it is difficult for stakeholders to compare their goals and assess whether the actions undertaken are sufficient to achieve a net neutral global economy by 2050. Moreover, it seems unrealistic to consider achieving carbon neutrality through abrupt emissions reductions alone, without emissions alone, without financing the transition to a sustainable economy. This is where the objectives of a net zero policy go beyond simply decarbonizing the system. We must therefore define such a policy by taking into account its two dimensions: the decarbonization of the system and the financing of its transition. This actuarial thesis aims to propose a practical implementation of a net zero policy for asset managers. First, we need to define the net zero scenario to be followed. This scenario is generally translated into a decarbonization trajectory for the year 2050. The reduction of carbon emissions is thus punctuated by an initial reduction $\boldsymbol{\mathcal{R}}^{-}$ as well as an annual reduction rate $\Delta\boldsymbol{\mathcal{R}}$ according to the following formula:

$$\boldsymbol{\mathcal{R}}\left(t_0, t\right) = 1 - \left(1 - \Delta\boldsymbol{\mathcal{R}}\right)^{t-t_0}\left(1 - \boldsymbol{\mathcal{R}}^{-}\right)$$

where t_0 is the base year, t the target year and $\boldsymbol{\mathcal{R}}\left(t_0, t\right)$ the reduction rate between t_0 et t.

Figure 75: Examples of decarbonization pathway (in %)



The net zero scenarios can thus differ in the choice of reduction parameters but also in the definition of the carbon emissions to be reduced. A company can choose to reduce its emissions in absolute terms or relative to a normalization variable. This measure of emissions, called carbon intensity, is defined by a ratio between the absolute emissions $\boldsymbol{\mathcal{CE}}\left(t\right)$ and the normalization variable $Y\left(t\right)$:

$$\boldsymbol{\mathcal{CI}}\left(t\right) = \frac{\boldsymbol{\mathcal{CE}}\left(t\right)}{Y\left(t\right)}$$

The reduction in intensity thus makes it possible to measure the emissions and efforts of a company relative to its size (market capitalization, turnover, etc.) and offers the possibility of comparing companies of different sizes or sectors. In addition, absolute emissions can be broken down into different categories. These "scopes" range from direct greenhouse gas emissions from sources owned and controlled by the issuer (scope 1) to indirect emissions from the distribution of the issuer's products through their consumption by customers and lifecycle (scope 3 downstream). The two scenarios mainly implemented in the financial market are the Climate Transition Benchmark (CTB) and Paris Aligned Benchmark (PAB). Both correspond to an annual reduction in carbon intensity of 7%, but differ in their initial reduction of 30% and 50% respectively. There are still many discussions about the scope of emissions to be taken into account in these net zero scenarios and we explain this by the lack of reliability and transparency of the available data. In particular, this brief illustrates the difficulty of including emissions from the entire value chain of the emitter (downstream scope 3) by showing the instability of data depending on the providers.

Another criticism directed at these intensity-based decarbonization scenarios is their lack of physical basis because the International Energy Agency's (IEA) carbon neutrality scenarios are actually constructed in terms of a carbon budget. They define a greenhouse gas limit in $GtCO_2e$[58] to be respected by 2050. If at first sight, carbon neutrality in intensity is equivalent to neutrality in emissions, we show that, under our assumptions, the BAP and BTC trajectories are more aggressive than the IEA scenarios. On the other hand, this paper explores different ways to direct investments towards companies that participate in the transition to a low-carbon economy. Unlike emissions estimation, the measurement of an emitter's ability to transition fails to become widespread. Indeed, it is sometimes difficult for a company to be a player in the transition while at the same time being key to the economy, particularly in its sector of activity, such as the health sector. This illustrates the need but also the difficulty to define simple measures of the "green intensity" of an issuer. In the context of this thesis, we present and refer to the European green taxonomy built by the european Technical Expert Group (TEG), which is still under development. The objective of this taxonomy is to define a set of activities defined as green and to allow companies to report a green revenue $\boldsymbol{\mathcal{GRS}}$ from these activities. From this metric, we define the green intensity of an the share of its revenue coming from these activities:

$$\boldsymbol{\mathcal{GRS}} = \frac{\boldsymbol{\mathcal{GR}}}{\boldsymbol{\mathcal{TR}}}$$

Finally, to complete this metric and measure the effort and capacity of an issuer to reduce its emissions, we regress its emissions history over time:

$$\boldsymbol{\mathcal{CE}}\left(t\right) = \beta_0 + \beta_1 \cdot t + u\left(t\right)$$

and derive an idiosyncratic decarbonation coefficient β_1. We normalize this coefficient with respect to the last absolute emissions of the issuer for the sake of comparison between different firms:

$$\boldsymbol{\mathcal{CM}}^{\mathcal{L}ong}\left(t\right) = \frac{\hat{\beta}_1\left(t\right)}{\boldsymbol{\mathcal{CE}}\left(t\right)}$$

The role of this coefficient is to identify the participants most likely to make efforts based on the evolution of their past emissions and to include them in our portfolios. The aim is not only to avoid excluding major emitters that are significantly reducing their emissions, but also to build a portfolio with a satisfactory level of self-decarbonation. This would allow us to finance a portfolio of companies that are on the path to carbon neutrality at the desired pace without having to make major changes to our positions.

Methodology

The literature now considers climate risk as an ordinary risk factor that is embedded in the traditional tools of financial institutions. It is now a question of measuring this risk in business-as-usual portfolios in order to study their capacity to integrate climate risk and to make the necessary modifications to align them with net-zero policies. In the end, this approach reflects the willingness of financial actors to modify their portfolios at the margin in order to bring them into line with the new regulatory constraints, rather than considering climate risk or carbon neutrality as an investment framework in its own right. In order to translate this desire for proximity between an aligned portfolio and business-as-usual, this thesis borrows the Markowitz portfolio construction methodology in the presence of a benchmark portfolio. The objective is to build a portfolio that is close to the benchmark portfolio, which here represents an example of a portfolio managed by a financial institution, but that respects various climate constraints defined with the help of the metrics put forward in this paper (carbon intensity, green revenue share, self-decarbonation, etc.).

[58] carbon dioxide equivalent in giga tons

Consider a universe of n investable assets. Let $b = (b_1, \ldots, b_n)$ be the vector of weights of each asset in the benchmark portfolio. In the context of this paper, the reference portfolio b invests in each asset in proportion to their market capitalization. We denote $R = (R_1, \ldots, R_n)$ the vector of asset returns in the universe where R_i is the return on asset i. The expected return of the reference portfolio is then :

$$\mu(b) = \mathbb{E}\left[b^\top R\right] = b^\top \mu$$

With $\mu = \mathbb{E}[R]$, while its variance is :

$$\sigma^2(b) = b^\top \Sigma\, b$$

with $\Sigma = \mathbb{E}\left[(R-\mu)(R-\mu)^\top\right]$ the covariance matrix of asset returns.

We now look for a portfolio x close to the benchmark portfolio b. A traditional measure of the closeness of these two portfolios is the replication error, or tracking error. If we denote $\mu\left(x \mid b\right) = (x-b)^\top \mu$ the excess return of portfolio x, we can define the volatility of the tracking error of portfolio x with respect to b by :

$$\sigma\left(x \mid b\right) = \sqrt{(x-b)^\top \Sigma (x-b)}$$

The objective is then to minimize this error.

Equity portfolio construction

We combine various portfolio construction constraints with this minimization problem as well as the climate constraints of decarbonization and transition[59]. The problem writes then as :

$$\begin{aligned} x^\star(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^\top \Sigma(t)\left(x - b(t)\right) \\ &\text{s.t.} \left\{ \begin{array}{ll} \boldsymbol{\mathcal{CI}}(t,x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0,t)\right)\cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonation} \\ x \in \Omega_{\boldsymbol{\mathcal{T}}ransition}(t) & \longleftarrow \text{Transition} \\ x \in \Omega_1 \cap \Omega_2(t) & \end{array} \right. \end{aligned} \tag{95}$$

where decarbonation follows the trajectories imposed by the BAP or BTC labels and the transition is defined by the set $\Omega_{\boldsymbol{\mathcal{T}}ransition}(t)$. In particular, we define this set by:

$$x \in \Omega_{\boldsymbol{\mathcal{T}}ransition}(t) \Leftrightarrow \left\{ \begin{array}{ll} \boldsymbol{\mathcal{GI}}(t,x) \geq \left(1 + \boldsymbol{\mathcal{G}}(t)\right)\cdot \boldsymbol{\mathcal{GI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{Greenness} \\ \boldsymbol{\mathcal{CM}}^{\mathcal{L}ong}(t,x) \leq \boldsymbol{\mathcal{CM}}^\star & \longleftarrow \text{Momentum} \end{array} \right. \tag{96}$$

Finally, the set $\Omega_1 \cap \Omega_2(t)$ defines traditional portfolio construction optimization constraints (long only portfolio, sectoral deviation limit, deformation limit with respect to the benchmark portfolio etc.).

Conclusions

Borrowing from a traditional portfolio construction approach that aims to minimize replication error, this actuarial dissertation provides the following insights into the implementation of net zero portfolios.

[59] The problem is adapted in case of a bond portfolio

Sensitivity to the emissions calculation method The first lesson concerns the sensitivity of the solution to parameters and data. In particular, fund managers must be careful when choosing the carbon emissions perimeter they use to decarbonize their portfolios. Net zero only makes sense if it covers all the emissions of a closed system. Therefore, it is the scope 3 emissions that must be taken into account to align a portfolio with a carbon neutrality scenario. The problem is that today we observe a lack of reliability around these emissions data. Nevertheless, it is important that financial actors (managers, regulators etc.) start using them in order to create incentives to improve their quality. However, this paper shows that the inclusion of scope 3 emissions increases the risk of replication error, especially for upstream emissions. Also, the resulting portfolio is highly dependent on the target we wish to achieve in terms of green revenue share and level of self-decarbonation. Fund managers must therefore be cautious because too much ambition in the short term implies that there may be no medium-term solution to the optimization problem, and therefore no alignment possible.

The distinction between decarbonization and alignment The second key result is that decarbonization and portfolio alignment yield different solutions because they are two distinct investment policies. In particular, decarbonizing a portfolio is easier than aligning it. In particular, we show that decarbonization according to the CTB or PAB trajectories never leads to an explosion of tracking errors until 2030. In fact, the real challenge of the decarbonization exercise lies in the diversification and liquidity risks that an investor may face. These risks are amplified when we add the transition dimension to the optimization program. Beyond a higher tracking error cost, there is even no guarantee that a solution still exists. Furthermore, the introduction of the transition pillar highlights the difficulty of choosing an appropriate set of constraints for net zero portfolios, as some parameters may be negatively correlated with others. Our results are furthermore in line with the literature: portfolio decarbonization is systematically a long strategy on financial issuers and short on energy, materials and utilities issuers. As a result, we find ourselves in a situation in which the transition capacity of a decarbonized portfolio is lower than that of the benchmark portfolio, as green solutions are also located in carbon-intensive sectors. It is therefore crucial to distinguish between high-carbon-footprint issuers that will not participate in the transition and those that will reduce their emissions and find sustainable solutions. Furthermore, because the transition is multifaceted, there is a temptation for practitioners to multiply the metrics for measuring the ability of a portfolio to support it. Unfortunately, this is not a reliable approach because these metrics can be independent in the short term (such as a company's self-decarbonation and green revenue share). Thus, it is best to focus on a small number of transition constraints and understand the purpose of each. A concise problem for defining net zero is more useful than a complex framework and diffuse stacking of criteria.

Diverging from business-as-usual The third key finding is that decarbonization and portfolio alignment are both exclusionary processes. This means that it is quite impossible to achieve net-zero alignment without allowing some assets to be excluded from the benchmark portfolio. As a result, some key transition players such as energy and utility companies suddenly disappear from the investable universe. Therefore, imposing proximity on a benchmark portfolio through sector neutrality can raise solution existence problems. The exclusion process we observe at both the issuer and sector level then raises a calibration issue. Indeed, while portfolio decarbonization can be seen as a slight tilt of the benchmark portfolio, portfolio alignment can involve a significant narrowing of the investment universe. As such, this narrowing shows that the definition of net-zero investment is complex because it is far too far from the usual investment. In the short term, the gap remains significant, and the choice to move away from business-as-usual methods is an important issue for all net zero investors.

New Approaches The final point concerns the implementation of net zero investment policies. In this dissertation, we have focused on the traditional top-down approach for its simplicity in obtaining quantitative results. However, it is not the only approach that can be considered. In particular, active management makes a lot of sense when implementing a net zero investment. For example, we presented the core-satellite approach, which includes the decarbonization dimension for the core investment (core) and the transition dimension for the satellite strategy. This framework is easier to implement than the integrated optimization approach that this dissertation explores. Moreover, it allows for control over the distribution between the two dimensions, and for the weight of the transition dimension to be gradually modified as the economy becomes greener. Currently, net zero could ultimately be considered a thematic investment because the universe of transition assets is still small. But in the future, if there is still a difference between net zero and traditional investment, it would mean that we have collectively failed to limit global warming.