Macroprudential policies and energy transition

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Abstract

Climate change is one of the most urgent challenges for humanity, requiring a smooth and controlled transition to a low-carbon economy. The literature has highlighted the importance of building a portfolio of climate policies that can be pursued jointly. Then, this paper looks at the effects of a macroprudential policy acting on the credit constraint in order to reallocate resources in favor of non-polluting capital. To do this, we use a model composed of heterogeneous agents with infinite lifetimes, of two sectors, the polluting and the non-polluting sector, and of a macroprudential policy in the form of a credit constraint. We then show that this policy has a positive effect on the redirection of financing towards the non-polluting sector, and that this effect is all the stronger when the technology of the non-polluting sector is less efficient than that of the polluting sector. We also show that this policy does not allow for a complete energy transition, and that it must be coupled with other climate policies in order to avoid a situation of environmental no-return.

1 Introduction

"195 Nations Set Path to Keep Temperature Rise Well Below 2 Degrees Celsius" - Historic Paris Agreement on Climate Change (Press Release) – 2015

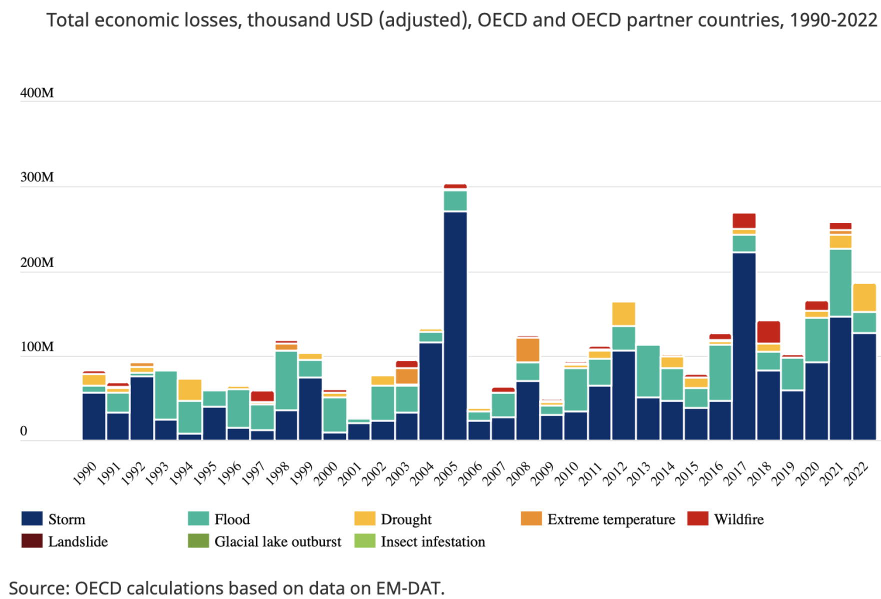

Figure 1: Total economic losses, thousand USD (adjusted), OECD and OECD partner countries, 1990-2022

The last four decades have seen a significant increase in the number of natural disasters worldwide, leading to growing economic losses. Forecasts point to rising temperatures and weather conditions continuing to worsen in a non-linear fashion, with irreversible consequences for the environment and the economy. Indeed, the study conducted by the World Meteorological Organization (WMO) shows a significant increase in economic losses linked to natural disasters over the last fifty years. Average daily losses have risen from 49 million dollars in the 1970s to 383 million dollars in 2010. Hurricanes Harvey, Maria and Irma in 2017 were among the most costly disasters, accounting for 35% of total global economic losses between 1970 and 2019. The OECD's Climate Action Monitor 2023 also indicates that economic losses due to natural disasters in 2022 have been estimated at $270 billion, with insured losses of around $120 billion, one of the highest on record.

Launched following the Paris agreements in 2015, the energy transition now appears to be a major economic, social and societal challenge, with the potential to generate high social costs if it is not carried out optimally.

On 29 September 2023, Christine Lagarde spoke on this subject at the international conference "Ensuring an orderly energy transition: Europe's competitiveness and financial stability in a period of global energy transformation", highlighting three elements "essential to a smooth transition: avoiding procrastination, understanding the challenges, and sharing the burden fairly".

This speech highlights the need to understand the challenges and limits associated with the energy transition so that it can be carried out in a controlled manner.
One of these challenges is to take account of financial frictions, and more specifically those relating to credit constraints. This type of information asymmetry hinders the redirection of funding towards the non-polluting sector.

Several theoretical studies, including **Acemoglu and al. (2012), Golosov and al. (2009)**, or **Gerlagh, Kverndokk and Rosendahl (2009)**, have shown how important it is for institutions to have a range of tools to deal effectively with environmental emergencie, and to allow a smooth transition. In addition, all the policies put in place address a specific risk. One of these risks, as the research carried out by **Carattini and al. (2022)** shows, is the presence of financial frictions when a climate policy is put in place. The presence of asymmetric information has also been widely studied, as shown by the research of **Mian and Sufi (2009)**, **Allen and Gale (1994)** or **Bernanke, Gertler and Gilchrist (1994)**. Finally, although the literature on this subject is very thin, it does offer a basis for classifying polluting and non-polluting assets.

Then, although the authorities already have a number of tools for effectively redirecting funding towards greener, environmentally-friendly technologies, namely taxes (carbon tax) and subsidies, we note that the literature does not offer policy tools to reduce the effects of financial frictions and support the energy transition.

In this paper, we design a model that highlights the effectiveness of macroprudential policies as a support on energy transition by incorporating a credit constraint. The main contribution of this paper, in the light of the existing literature, is based on the integration of a credit constraint enabling financing to be redirected flexibly towards the non-polluting sector. Furthermore, this tool provides a certain flexibility in its implementation, which Pigouvian tax policies do not allow.

To do this, we consider a dynamic general equilibrium model composed of agents with infinite lifetimes. The agents in this economy are heterogeneous. Households, considered as patient, will save, whereas entrepreneurs, considered as impatient here, will invest in two types of capital : polluting and non-

polluting. Moreover, through their willingness to borrow, these entrepreneurs will face imperfections in the credit market, which will be modelled by a collateral constraint.
Generally speaking, this model is in line with the RBC models in its macroeconomic structure, incorporating pollution that will affect the well-being of agents.

Several notable results can be highlighted. This paper shows that, compared with a model without macroprudential policy (modelled by an unsaturated credit constraint), we observe a significant increase in the share of non-polluting energy in equilibrium. The effectiveness of this policy also depends on the efficiency of the technology specific to the non-polluting sector. The less efficient the technology, the greater the effect of the macroprudential policy in redirecting financing to the non-polluting sector.
Furthermore, our simulations also show that implementing this policy will not prevent a situation of environmental no-return. It therefore needs to be coupled with other climate policies.

This paper will be divided into 10 sections. Section 2 reviews the related literature. Section 3 presents the model used. Section 4 is related to the research of the equilibrium. Section 5 focuses on the the static comparison at the balance growth path. Section 6 includes the calibration of the parameters for the macroeconomic and environmental variables that aim to fit some features of the observed data for the US economy. Section 7 presents the differences with the case when we have an unsaturated credit constraint. Section 8 discusses how the value of q influences the number of trajectories in the economy. Section 9 shows that the implementation of this policy can not prevent from an environmental disaster. Section 10 concludes and proposes some extensions.

2 Literature Review

2.1 The need for effective climate policies

The primary motivation for this research paper is to look at the construction of a macroprudential policy to avert an environmental disaster. Consequently, it is necessary to take an interest in the various studies on the integration of environmental constraints in the models, to understand the extent to which the implementation of appropriate policies can prevent such a situation from arising.

Environmental constraints were taken into account fairly early on. This is particularly true of **Solow (1974)** who presented an economic vision of the environmental constraint by showing that it is possible to reconcile economic growth and preservation of the environment. One of the pioneering models in

this field is **Nordhau's DICE model (1994)**, which incorporates climate change into its framework after working on an approach to aggregate damages in 13 different regions across the world.
At first, most of the models created focus on commutable general equilibrium with exogenous technology. This is notably the case of **Golosov and al. (2009)** who determined the structure of an optimal policy to be implemented in the case of exhaustible resources in a context of exogenous technologies. More in line with recent literature and no longer considering technology as an exogenous factor, **Acemoglu and al. (2012)** present a model of growth that incorporates environmental constraints as well as endogenous shocks due to the fact that resources may run out. In their work, pollution will affect the well-being of agents, as in the model presented in this paper but another way of incorporating pollution-related externalities is to include them in the TFP, along the lines of the research carried out by **Heutel (2012)**.These works also give recommendations to political interventions. This body of work suggests that growth and the integration of environmental constraints are possible, although this assertion has been the subject of debate, particularly in the work of **Stokey (1998)**, who announced that the environmental constraint constitutes an endogenous limit to growth.
However, for growth to be possible while integrating environmental constraints, the implementation of policies is essential. Indeed, the work of **Acemoglu and al. (2012)** advocates the introduction of subsidies coupled with a taxation system and **Gerlagh, Kverndokk and Rosendahl (2009)** have completed this idea by pointing out that the use of research subsidies would make it possible to reduce carbon taxes.
Furthermore, researchs lead by **Carattini, Heutel and Melkadze (2021)** confirms these elements. The authors state that a macroprudential policy without a carbon tax will have very little effect.

The first part of this literature review highlights the need for institutions to develop a range of tools to deal effectively with environmental emergencies and reduce the risks associated with this transition phase.
Moreover, this paper is in line with RBC models such as the model proposed by **Heutel (2012)**, while integrating the negative externalities linked to pollution into the welfare function of agents, as done by **Acemoglu and al. (2012)** in their paper. It is therefore a question of developing a macroprudential policy, to be implemented in conjunction with other climate policies, as is emphasised in most of the literature.

2.2 Financial frictions as a justification for a macroprudential policy

As this research paper focuses on the effects of implementing macroprudential policy, it is necessary to justify its legitimacy. This emerges when we look at the presence of financial frictions and their effects.

Information asymmetries between borrowers and lenders represent a major facet of financial frictions and a major challenge in financial markets, affecting credit conditions and economic dynamics. Empirical research by **Mian and Sufi (2009)** has highlighted the impact of these asymmetries on credit allocation, revealing how information imbalances can lead to restrictive credit conditions, despite the legitimacy of borrowers. This dynamic was also explored by **Allen and Gale (1994)**, who examined how information asymmetries can lead to inefficiencies in financial markets and distortions in the allocation of resources. Furthermore, the work of **Stiglitz and Weiss (1981)** has also made it possible to understand how information asymmetries can lead to credit rationing, even in the presence of solvent borrowers.
Thus, we have to focus on a specific part of the information asymmetries : the credit constraint.
Indeed, credit constraints, as a major impediment to access to finance for many economic actors, exert a significant influence on macroeconomic dynamics. The pioneering work of **Bernanke, Gertler and Gilchrist (1994)** highlighted the way in which these constraints can amplify the effects of economic shocks by triggering negative feedback mechanisms through the financial markets. This amplification of shocks was studied in depth by **Kiyotaki and Moore (1997)**, who explored how credit constraints can lead to self-perpetuating credit cycles, thereby amplifying economic fluctuations. In addition, research by **Cúrdia and Woodford (2015)** has examined how credit constraints can affect optimal monetary policy, highlighting the importance of taking these constraints into account in macroeconomic management.

We can also mention the transaction costs associated with financial activities, which have a significant impact on macroeconomic dynamics. The work of **Gertler and Gilchrist (1994)** examined in detail how these costs can influence the dynamics of business cycles by amplifying the effects of financial shocks. This amplification of shocks was also studied by **Brunnermeier and Sannikov (2014)**, where they examined how transaction costs can exacerbate macroeconomic volatility by amplifying the impact of financial shocks. Furthermore, research by **Holmström and Tirole (1997)** analysed how transaction costs can affect the efficiency of financial intermediation, highlighting their impact on the transmission of monetary policy and financial stability.

To complete the various sections above, we can cite other studies that have helped to understand the macroeconomic effects of financial frictions.

Indeed, the study conducted by **Brunnermeier, Eisenbach and Sannikov (2013)** showed that the presence of financial frictions in the system leads to the persistence of shocks, with significant non-linear amplification effects in a situation of illiquidity. The study by **Gertler and Kiyotaki (2010)** also shows that financial frictions can amplify economic fluctuations. The authors also highlight the fact that credit constraints can worsen recessions by restricting companies' access to the financing they need to maintain or develop their activities.

Consequently, the effects of these financial frictions cannot be neglected and lead us to question the type of climate policy that should be put in place to deal with them.

Financial frictions are thus a key element of our economies, as well as the financial stability as recent decades have shown. It is therefore necessary to understand the interactions between these financial frictions and the different types of climate policy in order to build a suitable policy model. Indeed, as announced by **Carattini and al. (2022)**, a risk to the stability of the macro-financial system may arise when a climate policy is combined with the presence of financial frictions.
Based on a DSGE model of an economy with two key market failures, namely a climate externality and financial frictions, the research conducted by **Carattini, Heutel and Melkadze (2023)** showed that macroprudential policies can reduce the risk of a recession following a major climate policy. This study also highlighted two major conclusions concerning the limits and advantages of macroprudential policies. Macroprudential policies can help to support economic growth once climate policies have been implemented (see the work carried out by **Acemoglu and al. (2012)** on the introduction of taxes and subsidies). However, it is important to note that macroprudential policies alone are ineffective in addressing climate externality in the absence of a comprehensive climate policy, which is one of the limitations of our model. Furthermore, the study conducted by **Brunnermeier, Eisenbach and Sannikov (2013)** shows that the actions of institutions to limit financial frictions are not solely positive, as these actions introduce an additional fragility linked to price instability.

To conclude this section on financial frictions and their systemic effects, it should be noted that the literature has highlighted the need for macroprudential and others climate policies to be complementary (and not substitutable), as each type of policy makes it possible to compensate for a specific market failure. In this case, the objective of the model presented in this research paper is to overcome the financial frictions related to credit constraints in order to improve the financial stability while taking environmental issues into account.

2.3 The need for a classification between polluting and non-polluting assets

These notions are directly linked to the model presented in this thesis. Indeed, the construction of a macroprudential policy aimed at reducing financial frictions related to credit constraints while integrating environmental constraints requires a reflection on the classification of assets between green and brown. Before beginning the final part of this literature review, it should be noted that these issues are relatively recent and that indicators for differentiating between green and brown assets are still being developed.

With a view to classifying economic activities and identifying those that are sustainable, in 2018 the European Commission launched the European Green Taxonomy, which the EU Member States adopted in June 2020. This green label indicates that, while respecting minimum human rights and labour law guarantees, an economic activity contributes to at least one environmental objective set by the European Commission.

The ECB has also underlined its commitment to the environment, by including the notions of green and brown collateral accepted as collateral for refinancing outstandings allocated to commercial banks. The measure relates to the risk posed by brown assets, so commercial banks are being asked to provide more brown assets for a same amount of green assets if they wish to use them as collateral. As a result, the value of brown assets is falling.

However, despite all these measures, there are no universal classifications between brown and green assets, and the literature on this subject remains fairly thin. We can nevertheless cite the research of **André and al. (2022)** who developed a series of indicators as a reference. However, they conclude this study by indicating that an improved and harmonised framework for the provision of non-financial information is essential in order to conduct an accurate analysis and adequately monitor the financial sector's exposure to the impacts of climate change.

This is therefore an area for future research.

3 The model

The model presented is made up of four blocks, namely households, entrepreneurs and firms, which are in line with the work proposed by **Genna (2021)**, as well as the environmental block inspired by the work of **Acemoglu and al. (2012)**.

Macroprudential policy will be integrated into the entrepreneur's block in the form of a credit constraint.

3.1 Households

We consider an infinite-horizon discrete time economy. We will considers that representative household consumes goods and work to maximize its inter-temporal utility. The utility function is modelled as follows :

$$U(C_t, L_t, S_t) = \ln(C_t^H) - \chi \ln(L_t) + \sqrt{S_t}$$

Here, C_t, L_t, and S_t respectively represent the household consumption, the number of hours worked, and the quality of the environment at time t. The coefficient χ is a scale parameter that affects the disutility of labor supply.

Furthermore, the household aims to maximize its lifetime expected utility given by the following expression:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln(C_t^H) - \chi \ln(L_t) + \sqrt{S_t})$$

where E_0 represents the mathematical expectation of all variables' future values and $\beta \in (0, 1)$ represents the constant discount factor. This β tends to be high, indicating a relatively weak preference for immediate consumption over future consumption.

Moreover, the budget constraint specific to households is given by the equality:

$$C_t^H + D_t = w_t L_t + R_t D_{t-1}$$

The left-hand side of the equation represents household expenditures. It consists of the variable d_t, representing the stock of deposits at time t, and the variable C_t^H, which is consumption at time t. Therefore, the household will make a trade-off between consumption and deposits.

The right-hand side of the equation represents household incomes, which are composed of two elements. We have $w_t L_t$, representing income earned from labor, as well as $R_t D_{t-1}$, representing the return on deposits made in the previous period. Indeed, households derive income from supplying labor and capital to firms at rental rates w_t and R_t.

The utility maximization program of the representative household is then given by :

$$\forall t \quad \max_{C_t, L_t, d_t} \sum_{t=0}^{\infty} \beta^t (\ln(C_t^H) - \chi \ln(L_t) + \sqrt{S_t})$$

subject to:

$$C_t^H + D_t = w_t L_t + R_t D_{t-1}$$

The first order condition of this household bloc are given by, according to appendix 1 :

$$w_t \cdot L_t + R_t \cdot D_{t-1} = C_t^H + D_t \tag{4}$$

$$w_t = \chi \cdot \frac{C_t^M}{L_t} \tag{5}$$

$$1 = \frac{C_t^M}{C_{t+1}^M} \cdot R_{t+1} \cdot \beta \tag{6}$$

Equation (4) is the budget constraint of households. Equation (5) is the labor equation, determining the amount of labor the household is willing to offer, as a function of wages. The last characteristic equation of this bloc is equation (6), which is the Euler equation, allows us to link present consumption to future consumption in order to determine the intertemporal choices made by households.

3.2 Entrepreneurs

Similarly to the previous case, we consider an infinity of entrepreneurs in an infinite-horizon discrete time economy. We will consider that the representative entrepreneur consumes only goods to maximize their inter-temporal utility. The utility function is modeled as follows :

$$U(C_t^E, S_t) = \ln(C_t^E) + \sqrt{S_t}$$

where C_t^E represents the entrepreneur's consumption, and S_t represents the environmental quality. Moreover, the entrepreneur aims to maximize their lifetime expected utility given by the following expression:

$$E_0 \sum_{t=0}^{\infty} \gamma^t (\ln(C_t^E) + \sqrt{S_t})$$

where E_0 represents the mathematical expectation of all variables' future values and $\gamma \in (0, 1)$ represents the constant discount factor. Note that $\gamma < \beta$, which means that the entrepreneur is more impatient than the household, thus explaining why they borrow.

Furthermore, the budget constraint of the entrepreneurs is given by the equality :

$$C_t^E + I_t^c + I_t^d + R_t B_{t-1} = R_t^c K_t^c + R_t^d K_t^d + B_t$$

The left-hand side of the equation represents the entrepreneur's expenses. It consists of the variable C_t^E, which is consumption at time t, I_t^c representing investment in clean capital at time t, I_t^d representing investment in dirty capital at time t, and $R_t B_{t-1}$ which is the repayment of the loan made in period $t-1$ (with the amount B_{t-1}) at the rate R_t. Therefore, the entrepreneur will make a first trade-off between consumption and investments, as well as a second trade-off between investment in clean or dirty capital.

The right-hand side of the equation represents the entrepreneur's income, composed of two elements. We have b_t, which is the amount borrowed in period t, as well as the income derived from investments in clean and dirty capital at rates R_t^c and R_t^d.

The utility maximization program of the representative entrepreneur is given by:

$$\forall t \qquad \max_{C_t^E, K_t^c, K_t^d, B_t} \sum_{t=0}^{\infty} \gamma^t (\ln(C_t^E) + \sqrt{S_t})$$

subject to:

$$\text{(i) } C_t^E + I_t^c + I_t^d + R_t B_{t-1} = R_t^c K_t^c + R_t^d K_t^d + B_t$$
$$\text{(ii) } K_{t+1}^d = I_t^d + (1 - \delta) K_t^d$$
$$\text{(iii) } K_{t+1}^c = q * I_t^c + (1 - \delta) K_t^c$$
$$\text{(iv) } R_{t+1} B_t \leq \theta_c R_{t+1}^c K_{t+1}^c + \theta_d R_{t+1}^d K_{t+1}^d$$

Equation (i) restates the budget constraint mentioned earlier. Equations (ii) and (iii) represent the capital accumulation of dirty and clean capital respectively. Note that the capital stock experiences depreciation at a rate of δ per period. Furthermore, there is a difference in technology between the polluting and non-polluting sectors, this difference being given by the structural parameter q.

Finally, equation (iv) represents the credit constraint, through which macroprudential policy is applied, represented by the coefficients θ_c and θ_d with $\theta_c > \theta_d$. Indeed, the entrepreneur will not be able to take out a loan whose amount exceeds a proportion of clean and dirty capital.

The first order condition of this entrepreneurs bloc are given by, according to appendix 2 :

$$\theta_c R^C_{t+1} K^c_{t+1} + \theta_d R^d_{t+1} K^d_{t+1} = R_{t+1} B_t \tag{14}$$

$$\frac{1/q + \theta_c \frac{R^c_{t+1}}{R_{t+1}}}{R^c_{t+1} + (1-\delta)\frac{1}{q} - \theta_c R^c_{t+1}} = \frac{1 + \theta_d \frac{R^d_{t+1}}{R_{t+1}}}{R^d_{t+1} + (1-\delta) - \theta_d R^d_{t+1}} \tag{19}$$

$$C^E_t + [K^d_{t+1} - (1-\delta)K^d_t] + \left[\frac{K^c_{t+1} - (1-\delta)K^c_t}{q}\right] + R_t B_{t-1} = R^c_t K^c_t + R^d_t K^d_t + B_t \tag{20}$$

To conclude with this second bloc, the entrepreneur equilibrium is given by the equation (19) which is the no arbitrage condition between clean and dirty capital. This condition is essential, otherwise all the investment would go to the same sector. The equation (14) is also an essential equation for this blocs. It is the saturated borrowing constraint. The third characteristic equation is given by (20), which is the budget constraint in which we inject the law of motion of both kind of capital.

Corrolary 1 :
Note that the credit constraint is only saturated under certain conditions, namely :

1. The first borrowing constraint binding is given by $R^c_{t+1} + \frac{1-\delta}{q} > \frac{R_{t+1}}{q} > \theta_c R^c_{t+1}$.

2. The second borrowing constraint binding is given by $R^d_{t+1} + (1-\delta) > R_{t+1} > \theta_d R^d_{t+1}$.

3.3 Firm

All markets are assumed to be perfectly competitive.
The firm produces a single output denoted by Y_t obtained through the use of labor L_t and capital K_t. In addition, we introduce growth through labor productivity denoted A_t, which grows exogenously at the rate λ. This production takes the form of a Cobb-Douglas function denoted by:

$$Y_t = (A_t L_t)^{1-\alpha} \cdot K^\alpha_t \quad \text{with} \quad A_{t+1} = A_t \cdot (1+\lambda)$$

where α represents the output elasticity of capital.

Furthermore, the capital K_t itself is composed of two types of capital. The first type of capital is denoted as K_t^c and represents all non-polluting machines (the 'c' stands for clean). The second type of capital used is, on the contrary, denoted by K_t^d and represents all polluting machines (the 'd' stands for dirty). This capital K_t is represented through a CES function:

$$K_t = \left((K_t^c)^\sigma + (K_t^d)^\sigma\right)^{\frac{1}{\sigma}}$$

Note that the clean capital and the dirty capital are used in equal proportions. Moreover, the coefficient σ is the parameter of elasticity of substitution. The higher this parameter, the more substitutable the factors are between each other.

Finally, the firm wants to maximize its profits denoted by the difference between incomes and costs. Consequently, the maximization program of the representative firm is given by:

$$\max \pi = Y_t - w_t L_t - r_t^c K_t^c - r_t^d K_t^d$$

with

$$Y_t = (A_t L_t)^{1-\alpha} \cdot \left((K_t^c)^\sigma + (K_t^d)^\sigma\right)^{\frac{\alpha}{\sigma}}$$

Before computing the first-order condition, we can introduce a new notion which is a key element of this model :

$$\kappa_t = \frac{K_t^{c\sigma}}{K_t^{c\sigma} + K_t^{d\sigma}} \tag{22}$$

This kappa will be the central element of our analysis in order to quantify the transition to a less polluting economy. Indeed, the closer we are from 1, the higher the share of no polluting energy.

The first order condition of this firm bloc are given by, according to appendix 3 :

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{21}$$

$$R_t^d = \alpha(1-\kappa_t)\frac{Y_t}{K_t^d} \tag{23}$$

$$R_t^c = \alpha\kappa_t\frac{Y_t}{K_t^c} \tag{24}$$

Then, we can conclude that equation (21), (23) and (24) maximize the profit of the firm.
These equations represent respectively the wage, the rental price of the dirty capital, and the rental price of the clean capital.
Furthermore, equation (22) is also a characteristic equation of this bloc.

3.4 Environment

This section is in line with the one proposed by **Xepapadeas and al. (2023)**. Indeed, following the approach of **Tahvonen and Kuuluvainen (1991)** as well as **Bovenberg and Smulders (1995)**, they modeled environmental quality as a stock of natural capital which have regenerative capacity nature and depreciates due to pollution, which is itself computed according to the stock of polluting capital.

Then, the quality of the environment is governed by the following law of motion:

$$S_{t+1} = -\xi \cdot K_t^d + (1+\theta) \cdot S_t$$

where ξ is the rate of environmental degradation resulting from the use of dirty capital and θ is the rate of environmental regeneration.

4 Equilibrium

4.1 Market clearing conditions

To complete the system of equations characterizing the equilibrium composed of (4), (5), (6), (14), (19), (20), (21), (22), (23) and (24), we can introduce three market clearing conditions.
The first one concerns the goods market with :

$$Y_t = C_t^E + C_t^M + i_t^c + i_t^d \tag{25}$$

The second one announces that :

$$D_t = B_t \tag{26}$$

The last one gives :

$$L_t = 1 \tag{27}$$

4.2 System reduction

From the 3 blocs and the market clearing conditions, we then get 13 equations characterizing the equilibrium. By reworking this system of 13 equations, we get a reduced system of 7 equations with 7 variables. This reduced system is given by :

$$D_t = \beta R_t D_{t-1} \tag{28}$$

$$\begin{aligned} K_{t+1}^c \left(\frac{1}{q} - \frac{\theta_c R_{t+1}^c}{R_{t+1}} \right) + K_{t+1}^d \left(1 - \frac{\theta_d R_{t+1}^c}{R_{t+1}} \right) = \gamma \Big[& \left(R_t^c (1 - \theta_c) + \frac{1-\delta}{q} \right) K_t^c \\ & + \left(R_t^d (1 - \theta_d) + 1 - \delta \right) K_t^d \Big] \end{aligned} \tag{29}$$

$$\frac{\frac{1}{q} + \frac{\theta_c R_{t+1}^c}{R_{t+1}}}{R_{t+1}^c + \frac{1-\delta}{q} - \theta_c R_{t+1}^c} = \frac{1 + \frac{\theta_d R_{t+1}^d}{R_{t+1}}}{R_{t+1}^d + 1 - \delta - \theta_d R_{t+1}^d} \tag{30}$$

$$D_t = \frac{\theta_c R_{t+1}^C K_{t+1}^c + \theta_d R_{t+1}^d K_{t+1}^d}{R_{t+1}} \tag{31}$$

$$R_t^c = \alpha \kappa_t \frac{(K_t^c)^\sigma + (K_t^d)^\sigma}{K_t^c} \tag{32}$$

$$\frac{R_t^c}{R_t^d} = \frac{\kappa_t}{1 - \kappa_t} \frac{K_t^d}{K_t^c} \tag{33}$$

$$\kappa_t = \frac{(K_t^c)^\sigma}{(K_t^c)^\sigma + (K_t^d)^\sigma} \tag{34}$$

4.3 Analytical results

By introducing growth through the exogenous parameters A_t, the steady state will be a balanced growth path. Consequently, it is necessary to rewrite the system of 7 equations with 7 unknowns above to show the intensive variables, of the form :

$$x_t = \frac{X_t}{A_t}$$

with X_t being the variable we wish to deflate. In addition, we pose $(1+\lambda) = \frac{A_{t+1}}{A_t}$.

Note that in the above system, the variables to be transformed are K_t^c, K_t^d, and D because at BGP, these three variables grow at the same rate, which in this case is λ.

Once the previous system has been rewritten in intensive form, we find the following analytical results :

$$R^* = \frac{(1+\lambda)}{\beta} \tag{35}$$

$$R^{c*} = \frac{\frac{1}{q}\left(\frac{(1+\lambda)}{\gamma} - (1-\delta)\right)}{1 - \theta_c + \theta_c \frac{\beta}{\gamma}} \tag{36}$$

$$R^{d^*} = \frac{\frac{(1+\lambda)}{\gamma} - (1-\delta)}{1 - \theta_d + \theta_d \frac{\beta}{\gamma}} \tag{37}$$

To simplify the following expressions, we have :

$$\Omega^* = \frac{q\left(1 - \theta_c + \theta_c \frac{\beta}{\gamma}\right)}{1 - \theta_d + \theta_d \frac{\beta}{\gamma}} \tag{38}$$

$$\tilde{k}^{c*} = \frac{\alpha}{\left(1 + (\Omega^*)^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-\alpha}{\sigma}}} \cdot \frac{\Omega^*\left(1 - \theta_d + \theta_d \frac{\beta}{\gamma}\right)}{\frac{(1+\lambda)}{\gamma} - (1-\delta)} \tag{39}$$

$$\tilde{k}^{d*} = \frac{\left(\frac{1-\kappa^*}{\kappa^*}\right) \cdot R^{c*} \cdot \tilde{k}^{c*}}{R^{d^*}} \tag{40}$$

$$\kappa^* = \frac{1}{1 + (\Omega^*)^{\frac{\sigma}{\sigma-1}}} \tag{41}$$

Corrolary 2 :

These equations are the proof of the uniqueness and existence of this balance growth path.

4.4 Study of local dynamics

In order to study the local dynamics of this BGP, we used Taylor developments of order 1 with respect to the values of the stationary state, with the aim of determining a system of 2 equations with 2 unknowns. All the characteristic equations are given in Appendix 4. These equations allow us to get the following system of 2 equations with 2 unknowns :

$$\begin{cases} \Omega_{21}\left(\frac{\tilde{k}^c_{t+1}-\tilde{k}^{c*}}{\tilde{k}^{c*}}\right) + \Omega_{22}\left(\frac{\tilde{k}^d_{t+1}-\tilde{k}^{d*}}{\tilde{k}^{d*}}\right) = \Omega_{23}\left(\frac{\tilde{k}^c_{t}-\tilde{k}^{c*}}{\tilde{k}^{c*}}\right) + \Omega_{24}\left(\frac{\tilde{k}^d_{t}-\tilde{k}^{d*}}{\tilde{k}^{d*}}\right) (70) \\ \Omega_{25}\left(\frac{\tilde{k}^c_{t+1}-\tilde{k}^{c*}}{\tilde{k}^{c*}}\right) - \Omega_{26}\left(\frac{\tilde{k}^d_{t+1}-\tilde{k}^{d*}}{\tilde{k}^{d*}}\right) = \Omega_{27}\left(\frac{\tilde{k}^c_{t}-\tilde{k}^{c*}}{\tilde{k}^{c*}}\right) - \Omega_{28}\left(\frac{\tilde{k}^d_{t}-\tilde{k}^{d*}}{\tilde{k}^{d*}}\right) (71) \end{cases}$$

Posing $\tilde{K}^c_{t+1} = \frac{\tilde{k}^c_{t+1}-\tilde{k}^{c*}}{\tilde{k}^{c*}}$ and $\tilde{K}^d_{t+1} = \frac{\tilde{k}^d_{t+1}-\tilde{k}^{d*}}{\tilde{k}^{d*}}$, we can rewrite this system of equations as follows:

$$\begin{pmatrix} \Omega_{21} & \Omega_{22} \\ \Omega_{25} & -\Omega_{26} \end{pmatrix} \begin{pmatrix} \tilde{K}^c_{t+1} \\ \tilde{K}^d_{t+1} \end{pmatrix} = \begin{pmatrix} \Omega_{23} & \Omega_{24} \\ \Omega_{27} & -\Omega_{28} \end{pmatrix} \begin{pmatrix} \tilde{K}^c_{t} \\ \tilde{K}^d_{t} \end{pmatrix} \tag{72}$$

The Jacobian matrix therefore appears :

$$J = \frac{1}{-\Omega_{26}\Omega_{21} - \Omega_{25}\Omega_{22}} \begin{pmatrix} -\Omega_{26}\Omega_{23} - \Omega_{22}\Omega_{27} & -\Omega_{26}\Omega_{24} + \Omega_{22}\Omega_{28} \\ -\Omega_{25}\Omega_{23} + \Omega_{27}\Omega_{21} & -\Omega_{24}\Omega_{25} - \Omega_{28}\Omega_{21} \end{pmatrix}$$

The point of finding this matrix form is to calculate the eigenvalues of this Jacobian matrix, in order to determine the number of trajectories in the economy. To determine them, we need to do is carry out a calibration to determine the various parameters of this matrix.

5 Static comparison at the balance growth path

Concerning the the static comparison of this steady state, a number of points can be made, in particular concerning the impact of the parameters q, θ_c and θ_d on κ^* which represents the proxy for energy transition, the closer κ is to 1, the higher the share of non-polluting energy.

The last equation (41) shows us that an increase in the structural factor q, representing the efficiency of clean technologies in the non-polluting sector, leads to an increase of κ^* and therefore encourages the transition to a greener economy.

Similarly, the increase in θ_c will also promote and support the energy transition to a certain extent. Finally, the increase in θ_d will encourage investment in polluting sectors, and therefore act as a brake on the energy transition.

Consequently, although the macroprudential policy introduced in this model does not allow for a complete energy transition (given by $\kappa = 1$), it can, however, support it and encourage a redirection of investment towards non-polluting sectors. To achieve this, it is necessary to maximise θ_c and minimise θ_d.

These static comparisons at steady state can be represented by the following graphs. In addition, the python codes used to obtain them can be found in the appendices 5 and 6.

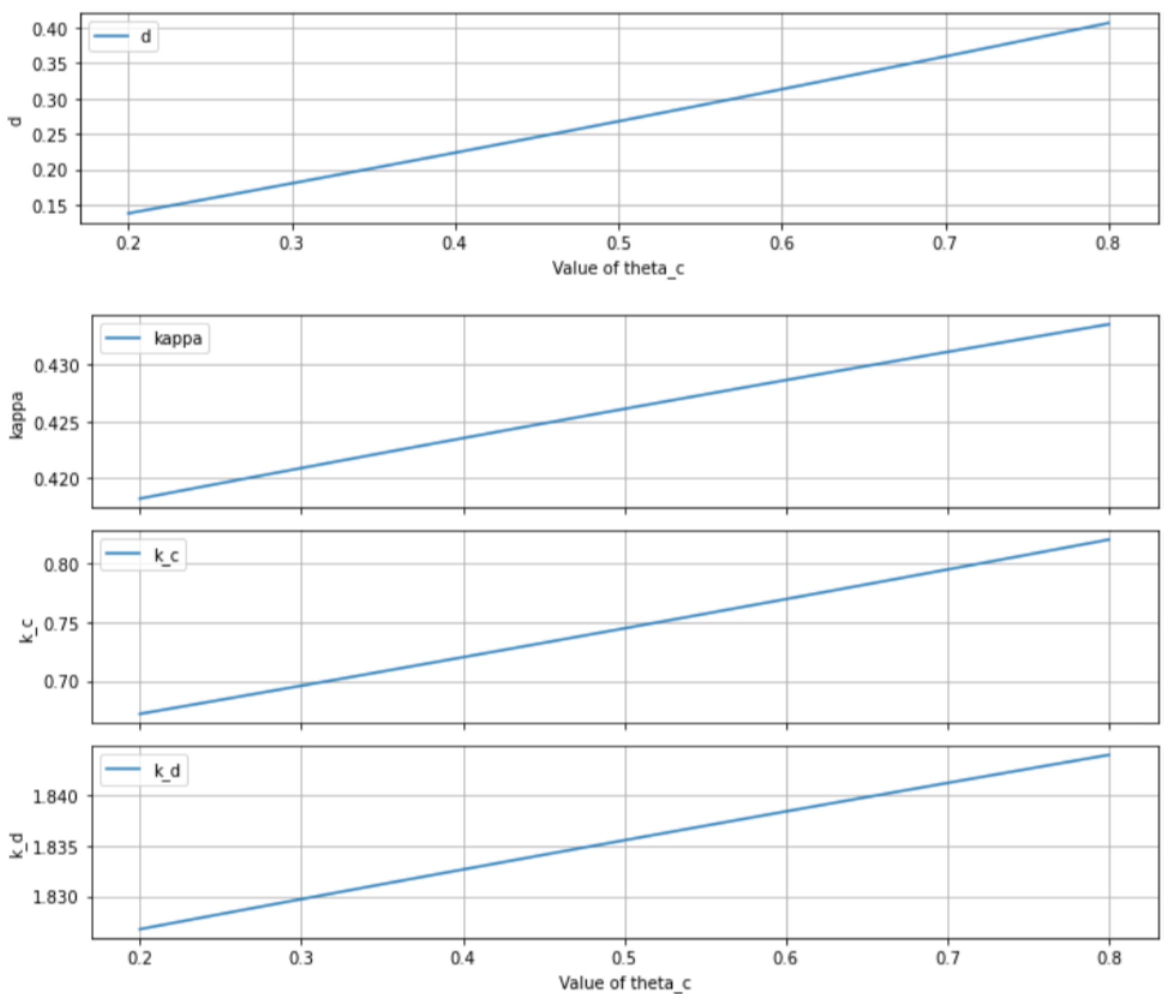

Figure 2: Effect of θ_c on the steady state

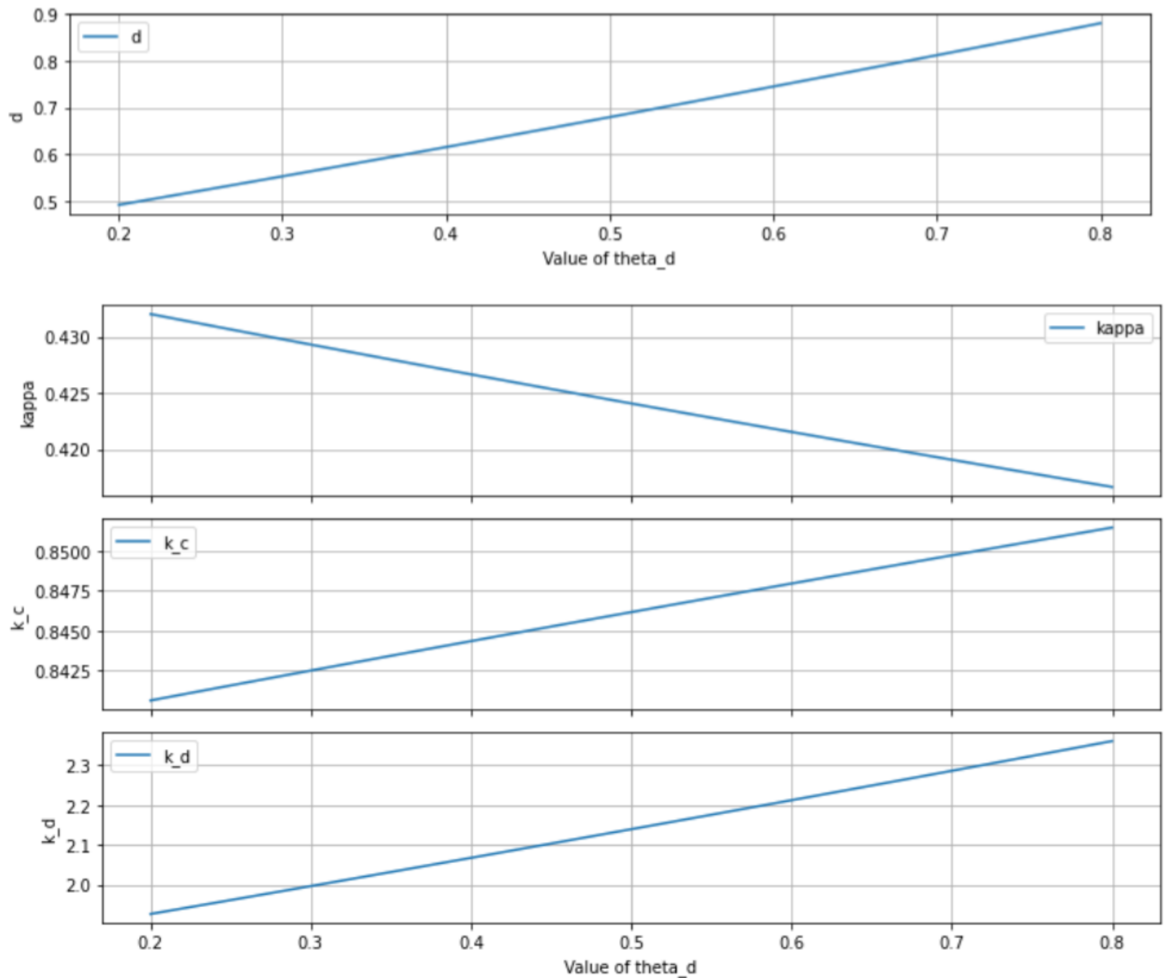

Figure 3: Effect of θ_d on the steady state

6 Calibration

6.1 Calibration of the macroeconomic variables

This calibration covers 9 main parameters, which are summarised in the table below. In line with RBC literature, we assume $\beta = 0.99$. As stated in the presentation of the model, entrepreneurs are considered to be less patient than households, so we arbitrarily assume $\gamma = 0.8$. According to **Papageorgiou, Saam and Schulte (2017)**, we fix $\sigma = 0.33$. The next parameter is α. Based on Bureau of Labor Statistics data and averaged from 2009 to 2019, then we get $\alpha = 0.4$. In relation to this same source, we will assume labor productivity growth at $\lambda = 1.5\%$ annually in the nonfarm business sector from 2007 to 2024. In addition, the depreciation rate of capital calibrated following **Barro and Sala-i-Martin (2004)**, the value $\delta = 0.05$ is widely used in macroeconomic literature.

To conclude with this part, we arbitrarily assume the following values: $q = 0.5$, $\theta_c = 0.85$, $\theta_d = 0.1$. Indeed, we set an initial condition of relative efficiency in the non-polluting sector such that $q < 1$. This calibration also respects the saturation conditions of the credit constraint given by corollary 1.

Parameter	**Value**	**Source**
β	0.99	RBC Literature
γ	0.8	Arbitrary assumption
σ	0.33	Papageorgiou et al. (2017)
α	0.4	Bureau of Labor Statistics (2009-2019)
λ	1.5 %	Bureau of Labor Statistics (2007-2024)
δ	0.05	Barro et Sala-i-Martin (2004)
q	0.5	Arbitrary assumption
θ_c	0.85	Arbitrary assumption
θ_d	0.1	Arbitrary assumption

Table 1: Summary of the macroeconomic variables

6.2 Calibration of the environmental variables

In the function concerning the law of motion of the environmental quality to study its evolution, we based our calibration on several studies that have been made. The rate of regeneration of the environmental stock fixed at $\theta = 0.04$ comes from the calibration carried out by **Nordhaus (1994)** and **Fullerton and Kim (2008)**. And finally, $\zeta = 0.17$ is set arbitrarily. However, we could have estimated this parameter empirically from the annual emission of CO_2 as done by **Acemoglu and al. (2012)**.

Parameter	**Value**	**Source**
θ	0.04	Nordhaus (1994), Fullerton et Kim (2008)
ζ	0.17	Arbitrary assumption

Table 2: Summary of the environmental variables

7 Unsaturated credit constraint and the effects of macroprudential policy

The interest of this paper is to show the impact of macroprudential policy compared with a situation in which no policy is implemented. Using the previous maximisation systems, the only difference concerns the block of entrepreneurs in which the Lagrange coefficient associated with the borrowing constraint is zero. All the calculations are presented in Appendix 7.

Note, however, that in this situation where the credit constraint is unsaturated, we obtain the following corollary.

Corrolary 3 :

In the case where the credit constraint is not saturated, we obtain the following relationship, which also enables us to determine the premium required for an investment in the non-polluting sector :

$$R_{t+1} = R^d_{t+1} + 1 - \delta = R^c_{t+1} + \frac{1+\delta}{q} \tag{73}$$

The following table shows the percentage differences between the kappa values obtained with and without the introduction of macroprudential policy. Note that we only vary the value of q and that we use the values given during calibration for $\theta_c = 0.85$ and $\theta_d = 0.1$. For each case, corollary 1 is verified. All simulations were carried out using dynare.

Value of q	**Value of κ^* with macroprudential policy**	**Value of κ^* without macroprudential policy**	**Percentage difference (from without to with introduction of the policy)**
0.5	0.43478	0.41547	+4.65%
1	0.519741	0.5	+3.95%
1.5	0.569231	0.5497	+3.55%
2	0.603581	0.584531	+3.25%
7	0.73856	0.722808	+2.18%

Table 3: Summary of κ values with and without macroprudential policy

Several elements can be highlighted from the results. Indeed they show that the introduction of

this macroprudential policy will have a positive impact on the value of kappa for every values of q, which indicates support for a less polluting economy.
We also observe that the lower the value of q, representing the technology associated with the non-polluting sector, the more effective macroprudential policy is. This is because agents will tend, through a substitution effect, to invest in the polluting sector when the value of q is low.

Consequently, the effects of this policy are positive and can be seen as supporting the energy transition. Furthermore, it is interesting to implement this policy early on (when q is less than 1), in order to have a greater positive effect on the redirection of capital towards the non-polluting sector.

8 The trajectory of the economy as a function of the q value

Our economy is made up of non-polluting capital and polluting capital, which represent the two predetermined variables of our model. Furthermore, section 4.4 gives us the Jacobian matrix, from which we can now calculate the eigenvalues, in order to determine the number of trajectories of this economy.

In the case where we take the values given during calibration with q = 0.5, we obtain two eigenvalues, both present in the unit circle. This allows us to say that there is a dynamic transition between the initial period and the long term. The economy is therefore converging towards the balance growth path.

However, we may ask whether this conclusion is similar for all values of q.

The simulations carried out using Python, the codes for which can be found in appendices 8 and 9, allow us to graphically represent the value of the two eigenvalues as a function of q.

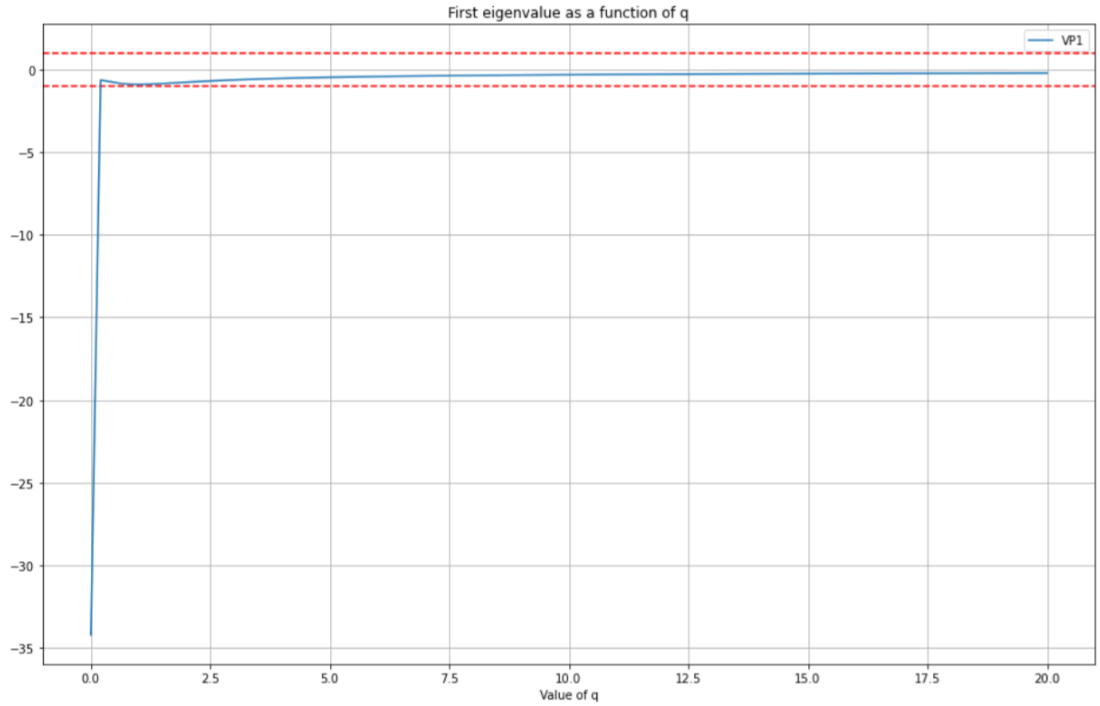

Figure 4: First eigenvalue

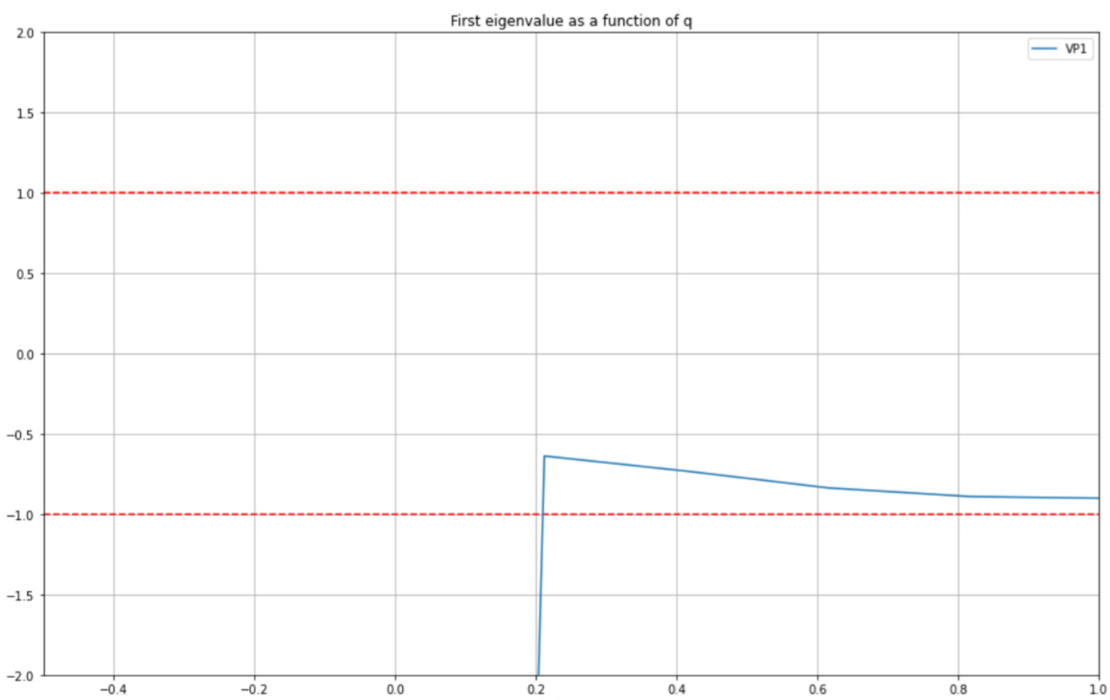

Figure 5: Zoom on the first eigenvalue

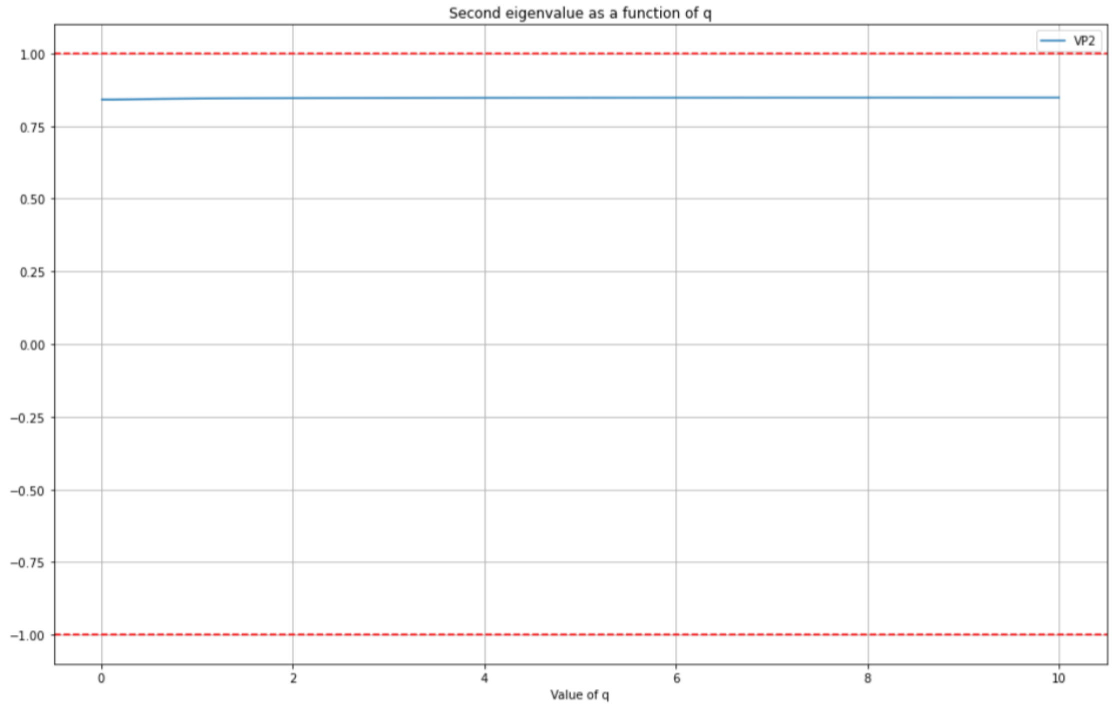

Figure 6: Second eigenvalue

As we can see from these graphs, one of the two eigenvalues is always included in the unit circle, but the other eigenvalue is only in the unit circle for certain values of q, in this case we must have approximately $q > 0.2$.
It will therefore be interesting to see whether this element can be verified empirically.

Consequently, if $q > 0.2$, our economy will converge towards the balance growth path and there is therefore only one trajectory representing the dynamic transition between the initial period and the long term. However, if $q < 0.2$, then our economy has no trajectory, indicating that there is no transition and that the economy must be directly on the balance growth path from the initial period.

9 A situation of environmental disaster

This section allows me to show the need to combine this macroprudential policy with other types of policy such as Pigouvian tax policies or subsidies. Indeed, we have seen that this type of macroprudential policy plays a supporting role in this energy transition by redirecting capital towards the non-polluting sector, but does not allow the stock of polluting capital to fall, which will be the source of an environmental disaster...

To do this, I have carried out simulations in which I place the initial conditions directly on the balanced growth path, where the stock of polluting capital grows at the same rate as labor productivity, which is $\lambda = 1.5\%$. We base ourselves on the initial values set out in the thesis by **Genna (2021)**, i.e., the initial stock of polluting capital is 15.1162 (value obtained from US data since 1970), as well as on the law of motion of environmental quality. Note also that $S_0 = 100 < \bar{S}$, which is the maximum level of environmental capital. Consequently, when the graph is displayed, it will be normal to see an initial increase in S_t, followed by a decrease in this stock.

The Python code used to obtain the following graph can be found in Appendix 10.

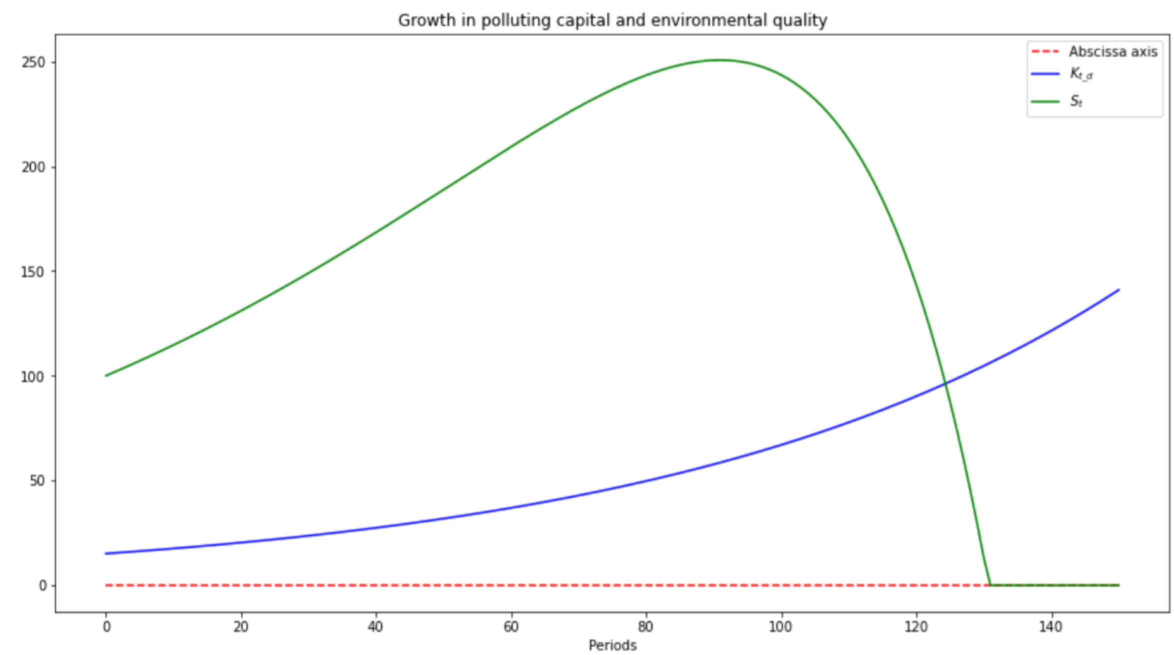

Figure 7: A situation of environmental disaster

As the graph shows, environmental quality (modelled by the green curve) rises when the stock of polluting capital (modelled by the blue curve) is relatively low. However, the growth of this polluting capital leads to an inflection point at a certain point in the stock of polluting capital, resulting in a significant drop in environmental quality until a point of no return is reached, modelled by S_t=0.
It is therefore necessary to implement several types of policy in a coordinated way to avoid reaching this threshold of no return.

10 Conclusion

Climate change is one of the most urgent challenges for humanity, and thus we need a clear understanding of its key challenges.

Based on the need highlighted in the literature to construct a portfolio of climate policies, and on the presence of financial frictions relating to information asymmetries and more specifically to credit constraints, this model looks at the implementation of a macroprudential policy to support the energy transition. The flexible nature of such a policy should also be emphasised, in comparison with Pigouvian tax policies, which have been the source of protests and uprisings.

To do so, we present a dynamic general equilibrium model composed of heterogeneous agents with infinite lifetimes. This model is composed of two sectors, the polluting and the non-polluting sector, allowing us to determine the effects of a macroprudential policy, appearing in the form of a credit constraint, on the redirection of financing towards the non-polluting sector. We also introduce the environmental dimension which will impact, as **Acemoglu and al. (2012)**, the well-being of agents directly in the utility function.

Several results can be highlighted.

Firstly, compared with a model in which the credit constraint is not saturated, we observe a significant positive effect of macroprudential policy on the redirection of financing towards the non-polluting sector. It should also be noted that this effectiveness depends on the technological efficiency of the non-polluting sector. Macroprudential policy is more effective when the technology of the non-polluting sector is weaker than that of the polluting sector, thereby countering the trade-off between these two sectors and supporting the energy transition.

Secondly, this technology relating to the non-polluting sector has an impact on convergence towards the balance growth path. We observe that if $q > 0.2$, the economy will converge towards the balance growth path. Conversely, if $q \leq 0.2$, then the economy must be placed on this BGP directly at the initial state.

Furthermore, when we look at the BGP, we can say that macroprudential policy can be seen as partial support for the energy transition because the value of the degree of pleagiability (θ_c and θ_d) will impact the value of κ^* but do not allow us to achieve an economy in which we only use non-polluting technologies modelled by $\kappa^* = 1$.

I would also like to stress in particular the fact that implementing this macroprudential policy does not make it possible, in view of the simulations carried out, to inhibit a situation of environmental no-return, which would be a dramatic situation. These elements confirm the research carried out by **Carattini, Heutel and Melkadze (2021)**, asserting the need to combine it with other types of policy.

Finally, this paper paves the way for a great deal of future research. The model presented relies on the ability to differentiate between a polluting and a non-polluting investment, which is still a very thin area of research. In addition, we will be able to determine the optimal values of θ_c and θ_d to maximise the well-being of our economy.

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11 Appendix

11.1 Household first order conditions

We notice that the optimization program of households has a recursive form, we can thus form a dynamic lagrangian as follows:

$$\forall t \quad L_t^H(C_t^M, L_t, S_t, D_t) = \beta^t \cdot E\left[\ln(C_t^M) - \chi \ln(L_t) + \sqrt{S_t}\right] + \lambda_t \left[w_t \cdot L_t + R_t D_{t-1} - C_t^M - D_t\right]$$

Where λ_t represents the Lagrange multiplier and E_t the expectation operator. We then solve the first-order partial derivatives of the above equation with regard to consumption, labor, deposits, and the Lagrange multiplier. We can get the four following First Order Conditions (FOC): Let $\frac{\partial L_t^H}{\partial C_t^M} = 0$, one can obtain the FOC with regard to households consumption:

1. Let $\frac{\partial L_t^H}{\partial C_t^M} = 0$, one can obtain the FOC with regard to households consumption:

$$\lambda_t = \beta^t \cdot \frac{1}{C_t^M} \tag{1}$$

2. Let $\frac{\partial L_t^H}{\partial L_t} = 0$, one can obtain the FOC with regard to labor:

$$0 = -\beta^t \cdot \chi \cdot \frac{1}{L_t} + \lambda_t \cdot w_t \tag{2}$$

3. Let $\frac{\partial L_t^H}{\partial D_t} = 0$, one can obtain the FOC with regard to deposits:

$$R_{t+1}^d = \frac{\lambda_t}{\lambda_{t+1}} \tag{3}$$

4. Let $\frac{\partial L_t^H}{\partial \rho_t} = 0$, one can obtain the budget constraint:

$$w_t \cdot L_t + R_t \cdot D_{t-1} = C_t^M + D_t \tag{4}$$

Combining (1) and (2), we get the labor equation:

$$w_t = \chi \cdot \frac{C_t^M}{L_t} \tag{5}$$

Furthermore, (3) and (1) allow us to write the Euler Equation:

$$1 = \frac{C_t^M}{C_{t+1}^M} \cdot R_{t+1} \cdot \beta \tag{6}$$

11.2 Entrepreneurs first order conditions

We notice that the optimization program of entrepreneurs has a recursive form, we can thus form a dynamic Lagrangian as follows:

$$
\begin{aligned}
\forall t \quad L_t^E(C_t^E, B_t, S_t, K_t^c, K_t^d) = \gamma^t[\ln(C_t^E) + \sqrt{S_t}]& \\
+ P_t[R_t^c K_t^c + R_t^d K_t^d + B_t - R_t B_{t-1} - C_t^E - i_t^c - i_t^d]& \\
+ \mu_t^c[q \cdot i_t^c + (1-\delta)K_t^c - K_{t+1}^c]& \\
+ \mu_t^d[i_t^d + (1-\delta)K_t^d - K_{t+1}^d]& \\
+ \eta_t[\theta_c R_t^C K_t^c + \theta_d R_t^d K_t^d - R_{t+1} B_t]&
\end{aligned}
$$

Where P_t, μ_t^c, μ_t^d, and η_t represent the Lagrange multipliers. Notice that at equilibrium, the borrowing constraint is saturated. We then solve the first-order partial derivatives of the above equation with regard to consumption, both types of investment (c and d), borrowing, and the Lagrange multiplier. We can get the following First Order Conditions (FOC) :

1. Let $\frac{\partial L_t^E}{\partial C_t^E} = 0$ one can obtain the FOC with regard to entrepreneurs' consumption:

$$P_t = \gamma^t \cdot \frac{1}{C_t^E} \tag{7}$$

2. Let $\frac{\partial L_t^E}{\partial i_t^c} = 0$ one can obtain the FOC with regard to investment in non-polluting capitals:

$$P_t = q \cdot \mu_t^c \tag{8}$$

3. Let $\frac{\partial L_t^E}{\partial i_t^d} = 0$ one can obtain the FOC with regard to investment in polluting capitals:

$$P_t = \mu_t^d \tag{9}$$

4. Let $\frac{\partial L_t^E}{\partial B_t} = 0$ one can obtain the FOC with regard to borrowing:

$$P_t - R_{t+1} \cdot P_{t+1} - \eta_{t+1} \cdot R_{t+1} = 0 \tag{10}$$

5. Let $\frac{\partial L_t^E}{\partial P_t} = 0$ one can obtain the FOC with regard to the first Lagrange multiplier, giving the

budget constraint of entrepreneurs:

$$C_t^E + i_t^c + i_t^d + R_t B_{t-1} = R_t^C K_t^c + R_t^d K_t^d + B_t \tag{11}$$

6. Let $\frac{\partial L_t^E}{\partial \mu_t^c} = 0$ one can obtain the FOC with regard to the second Lagrange multiplier, giving the law of motion of non-polluting capital:

$$q \cdot i_t^c + (1-\delta)K_t^c = K_{t+1}^c \tag{12}$$

7. Let $\frac{\partial L_t^E}{\partial \mu_t^d} = 0$ one can obtain the FOC with regard to the third Lagrange multiplier, giving the law of motion of polluting capital:

$$i_t^d + (1-\delta)K_t^d = K_{t+1}^d \tag{13}$$

8. Let $\frac{\partial L_t^E}{\partial \eta_t} = 0$ one can obtain the FOC with regard to the last Lagrange multiplier, giving the saturated borrowing constraint:

$$\theta_c R_{t+1}^C K_{t+1}^c + \theta_d R_{t+1}^d K_{t+1}^d = R_{t+1} B_t \tag{14}$$

9. Let $\frac{\partial L_t^E}{\partial K_{t+1}^c} = 0$ one can obtain the FOC with regard to the stock of clean capital at $t+1$:

$$P_{t+1} R_{t+1}^c - \mu_t^c + \mu_{t+1}^c (1-\delta) + \theta_c \eta_{t+1} R_{t+1}^c = 0 \tag{15}$$

10. Let $\frac{\partial L_t^E}{\partial K_{t+1}^d} = 0$ one can obtain the FOC with regard to the stock of polluting capital at $t+1$:

$$-\mu_t^d + R_{t+1}^d P_{t+1} + \mu_{t+1}^d (1-\delta) + \theta_d \eta_{t+1} R_{t+1}^d = 0 \tag{16}$$

Knowing (8), (9), and by isolating η_{t+1} in (10), we can then rewrite (15), giving us the first Euler Equation :

$$\gamma = \frac{C_{t+1}^E}{C_t^E} \cdot \frac{1/q + \theta_c \frac{R_{t+1}^c}{R_{t+1}}}{R_{t+1}^c + (1-\delta)\frac{1}{q} - \theta_c R_{t+1}^c} \tag{17}$$

Following the same reasoning, we can rewrite (16), giving us the second Euler Equation:

$$\gamma = \frac{C_{t+1}^E}{C_t^E} \cdot \frac{1 + \theta_d \frac{R_{t+1}^d}{R_{t+1}}}{R_{t+1}^d + (1-\delta) - \theta_d R_{t+1}^d} \tag{18}$$

From these two previous equations, we get the no arbitrage condition between clean and dirty capital, given by:

$$\frac{1/q + \theta_c \frac{R^c_{t+1}}{R_{t+1}}}{R^c_{t+1} + (1-\delta)\frac{1}{q} - \theta_c R^c_{t+1}} = \frac{1 + \theta_d \frac{R^d_{t+1}}{R_{t+1}}}{R^d_{t+1} + (1-\delta) - \theta_d R^d_{t+1}} \tag{19}$$

Furthermore, the last characteristic equation is given by (11) in which we inject (12) and (13) giving :

$$C^E_t + [K^d_{t+1} - (1-\delta)K^d_t] + \left[\frac{K^c_{t+1} - (1-\delta)K^c_t}{q}\right] + R_t B_{t-1} = R^c_t K^c_t + R^d_t K^d_t + B_t \tag{20}$$

11.3 Firms first order conditions

On the firm side, we can set the profit maximization problem as follows :

$$\forall t \quad L^F_t(L_t, K^c_t, K^d_t) = (A_t L_t)^{1-\alpha}\left(K^{c\sigma}_t + K^{d\sigma}_t\right)^{\alpha/\sigma} - w_t L_t - R^C_t K^c_t - R^d_t K^d_t \tag{1}$$

We then solve the first-order partial derivatives of the above equation with regard to labor and both types of capital stock (c and d).

We can now get the following First Order Conditions (FOC):

1. Let $\frac{\partial L^F_t}{\partial L_t} = 0$ one can obtain the FOC with regard to labor:

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{21}$$

2. Let $\frac{\partial L^F_t}{\partial K^d_t} = 0$ one can obtain the FOC with regard to the stock of polluting capital:

$$R^d_t = \alpha(1-\kappa_t)\frac{Y_t}{K^d_t} \tag{23}$$

3. Let $\frac{\partial L^F_t}{\partial K^c_t} = 0$ one can obtain the FOC with regard to the stock of no polluting capital:

$$R^c_t = \alpha\kappa_t \frac{Y_t}{K^c_t} \tag{24}$$

with

$$Y_t = (A_t L_t)^{1-\alpha} \cdot \left((K^c_t)^\sigma + (K^d_t)^\sigma\right)^{\frac{\alpha}{\sigma}}$$

and

$$\kappa_t = \frac{K^{c\sigma}_t}{K^{c\sigma}_t + K^{d\sigma}_t} \tag{22}$$

11.4 Study of the local dynamics

We assume the following relationships :

$$\Omega_1 = -\sigma\kappa^* - \alpha(1-\kappa^*) + \alpha \tag{42}$$

$$\Omega_2 = -\sigma\kappa^* - \alpha(1-\kappa^*) + 1 \tag{43}$$

$$\Omega_3 = \left(1 - \frac{\alpha}{\sigma}\right)\sigma(1-\kappa^*) + \alpha - 1 \tag{44}$$

$$\Omega_4 = \left(1 - \frac{\alpha}{\sigma}\right)\sigma(1-\kappa^*) \tag{45}$$

$$\Omega_5 = \frac{\theta_c R^{c*}}{R^*/q - \theta_c R^{c*}} - \frac{\theta_d R^{d*}}{R^* - \theta_d R^{d*}} \tag{46}$$

$$\begin{aligned}\Omega_6 = &\left(\frac{1-\theta_d}{R^{d*}(1-\theta_d)+1-\delta} + \frac{\theta_d}{R^*-\theta_d R^{d*}}\right) R^{d*}\left(\left(\frac{-\kappa^*}{1-\kappa^*} - \frac{\alpha}{\sigma}\right)\sigma(1-\kappa^*)+\alpha\right)\\ &-\left(\frac{1-\theta_c}{R^{c*}(1-\theta_c)+\frac{1-\delta}{q}} + \frac{\theta_c}{R^*-\theta_c R^{c*}}\right) R^{c*}\left(\left(1-\frac{\alpha}{\sigma}\right)\sigma(1-\kappa^*)\right)\end{aligned} \tag{47}$$

$$\begin{aligned}\Omega_7 = &\left(\frac{1-\theta_d}{R^{d*}(1-\theta_d)+1-\delta} + \frac{\theta_d}{R^*-\theta_d R^{d*}}\right) R^{d*}\left(\left(\frac{-\kappa^*}{1-\kappa^*} - \frac{\alpha}{\sigma}\right)\sigma(1-\kappa^*)+1\right)\\ &-\left(\frac{1-\theta_c}{R^{c*}(1-\theta_c)+\frac{1-\delta}{q}} + \frac{\theta_c}{R^*-\theta_c R^{c*}}\right) R^{c*}\left(\left(1-\frac{\alpha}{\sigma}\right)\sigma(1-\kappa^*)\right)\end{aligned} \tag{48}$$

$$\Omega_8 = \left(\frac{1}{q} - \theta_c\frac{R^{c*}}{R^*}\right)\tilde{k}^{c*} \tag{49}$$

$$\Omega_9 = \left(1 - \theta_d\frac{R^{d*}}{R^*}\right)\tilde{k}^{d*} \tag{50}$$

$$\Omega_{10} = \tilde{k}^{c*}\theta_c\frac{R^{c*}}{R^*} \tag{51}$$

$$\Omega_{11} = \tilde{k}^{d*}\theta_d\frac{R^{d*}}{R^*} \tag{52}$$

$$\Omega_{12} = \left(\tilde{k}^{c*}\theta_c R^{c*} + \tilde{k}^{d*}\theta_d R^{d*}\right)\frac{1}{R^*} \tag{53}$$

$$\Omega_{13} = \tilde{k}^{c*}\left(R^{c*}(1-\theta_c) + \frac{1-\delta}{q}\right) \tag{54}$$

$$\Omega_{14} = \tilde{k}^{d*}\left(R^{d*}(1-\theta_d) + 1 - \delta\right) \tag{55}$$

$$\Omega_{15} = \tilde{k}^{c*}R^{c*}(1-\theta_c) \tag{56}$$

$$\Omega_{16} = \tilde{k}^{d*}R^{d*}(1-\theta_d) \tag{57}$$

$$\Omega_{17} = \theta_c\frac{R^{c*}}{R^*}\tilde{k}^{c*} + \theta_d\frac{R^{d*}}{R^*}\tilde{k}^{d*} \tag{58}$$

$$\Omega_{18} = \theta_c\frac{R^{c*}}{R^*}\tilde{k}^{c*} \tag{59}$$

$$\Omega_{19} = \theta_d\frac{R^{d*}}{R^*}\tilde{k}^{d*} \tag{60}$$

$$\Omega_{20} = \left(\tilde{k}^{c*}\theta_c R^{c*} - \tilde{k}^{d*}\theta_d R^{d*}\right)\frac{1}{R^*} \tag{61}$$

$$\Omega_{21} = \frac{(1+\lambda)}{\gamma}\left(\Omega_8 - \Omega_{10}\Omega_3 - \Omega_{11}\Omega_1 + \frac{\Omega_{12}}{\Omega_5}\Omega_6\right) \tag{62}$$

$$\Omega_{22} = \frac{(1+\lambda)}{\gamma}\left(\Omega_9 + \Omega_{10}\Omega_4 + \Omega_{11}\Omega_2 - \frac{\Omega_{12}}{\Omega_5}\Omega_7\right) \tag{63}$$

$$\Omega_{23} = \Omega_{13} + \Omega_{15}\Omega_3 + \Omega_{16}\Omega_1 \tag{64}$$

$$\Omega_{24} = \Omega_{14} - \Omega_{15}\Omega_4 - \Omega_{16}\Omega_2 \tag{65}$$

$$\Omega_{25} = \frac{\Omega_{18}}{\Omega_{17}}(1+\Omega_3) + \frac{\Omega_{19}}{\Omega_{17}}\Omega_1 - \frac{\Omega_{20}}{\Omega_{17}}\frac{\Omega_6}{\Omega_5} \tag{66}$$

$$\Omega_{26} = \frac{\Omega_{19}}{\Omega_{17}}(1+\Omega_2) + \frac{\Omega_{18}}{\Omega_{17}}\Omega_4 - \frac{\Omega_{20}}{\Omega_{17}}\frac{\Omega_7}{\Omega_5} \tag{67}$$

$$\Omega_{27} = \frac{\Omega_{18}}{\Omega_{17}}(1+\Omega_3) + \frac{\Omega_{19}}{\Omega_{17}}\Omega_1 - \frac{\Omega_{20}}{\Omega_{17}}\frac{\Omega_6}{\Omega_5} + \frac{\Omega_6}{\Omega_5} \tag{68}$$

$$\Omega_{28} = \frac{\Omega_{19}}{\Omega_{17}}(1+\Omega_2) + \frac{\Omega_{18}}{\Omega_{17}}\Omega_4 - \frac{\Omega_{20}}{\Omega_{17}}\frac{\Omega_7}{\Omega_5} + \frac{\Omega_7}{\Omega_5} \tag{69}$$

11.5 Python code showing the impact of an increase of θ_c on the steady state

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve

# Definition du modele
def model_politique_macroprudentielle(x, beta, alpha, gamma, delta,
    theta_c, theta_d, sigma, lambd, q):
    d, R, R_c, R_d, kappa, k_c, k_d = x
    F = [
        -d + (beta / (1 + lambd)) * R * d,
        -R_c + alpha * kappa**(1 - alpha / sigma) * k_c**(alpha - 1),
        -R_d + (1 - kappa) / kappa * k_c / k_d * R_c,
        -((1 + lambd) / gamma) * (k_c * (1 / q - theta_c * R_c / R) +
            k_d * (1 - theta_d * R_d / R)) + (k_c * (R_c * (1 - theta_c)
            + (1 - delta) / q) + k_d * (R_d * (1 - theta_d) + 1 - delta))
            ,
        -d + (1 + lambd) * (theta_c * (R_c / R) * k_c + theta_d * (R_d /
             R) * k_d),
        -kappa + (k_c**sigma) / (k_c**sigma + k_d**sigma),
        -(R_d * (1 - theta_d) + 1 - delta) / (1 - theta_d * (R_d / R)) +
             (R_c * (1 - theta_c) + (1 - delta) / q) / (1 / q - theta_c *
             (R_c / R))
    ]
    return F

# Parametres initiaux
beta = 0.99
alpha = 0.4
gamma = 0.8
```

```python
delta = 0.05
theta_d = 0.1
sigma = 0.33
lambd = 0.016
q = 0.5

# Plage de valeurs de theta_c
theta_c_values = np.linspace(0.2, 0.8, num=30)

variable_values_all = []

for theta_c in theta_c_values:
    equilibrium_values = fsolve(model_politique_macroprudentielle, [1,
        1, 0.14, 0.3, 0.4, 3, 1], args=(beta, alpha, gamma, delta,
        theta_c, theta_d, sigma, lambd, q))
    variable_values_all.append(equilibrium_values)

variable_values_all = np.array(variable_values_all)

# Afficher l'evolution des variables d, kappa, k_c et k_d pour chaque
    valeur de theta_c
variables = ['d', 'kappa', 'k_c', 'k_d']
fig, ax = plt.subplots(figsize=(10, 2.5))

ax.plot(theta_c_values, variable_values_all[:, 0], label='d')
ax.set_ylabel('d')
ax.legend()
ax.grid()

ax.set_xlabel('Value of theta_c')
plt.tight_layout()
plt.show()

variables = ['kappa', 'k_c', 'k_d']
fig, axes = plt.subplots(len(variables), 1, figsize=(10, 6), sharex=True
    )

for i, ax in enumerate(axes):
```

```
    ax.plot(theta_c_values, variable_values_all[:, 4 + i], label=
        variables[i])
    ax.set_ylabel(variables[i])
    ax.legend()
    ax.grid()

axes[-1].set_xlabel('Value of theta_c')
plt.tight_layout()
plt.show()
```

Listing 1: Python code showing the impact of an increase of θ_c on the steady state

11.6 Python code showing the impact of an increase of θ_d on the steady state

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve

# Definition du modele
def model_politique_macroprudentielle(x, beta, alpha, gamma, delta,
    theta_c, theta_d, sigma, lambd, q):
    d, R, R_c, R_d, kappa, k_c, k_d = x
    F = [
        -d + (beta / (1 + lambd)) * R * d,
        -R_c + alpha * kappa**(1 - alpha / sigma) * k_c**(alpha - 1),
        -R_d + (1 - kappa) / kappa * k_c / k_d * R_c,
        -((1 + lambd) / gamma) * (k_c * (1 / q - theta_c * R_c / R) +
            k_d * (1 - theta_d * R_d / R)) + (k_c * (R_c * (1 - theta_c)
            + (1 - delta) / q) + k_d * (R_d * (1 - theta_d) + 1 - delta))
            ,
        -d + (1 + lambd) * (theta_c * (R_c / R) * k_c + theta_d * (R_d /
             R) * k_d),
        -kappa + (k_c**sigma) / (k_c**sigma + k_d**sigma),
        -(R_d * (1 - theta_d) + 1 - delta) / (1 - theta_d * (R_d / R)) +
             (R_c * (1 - theta_c) + (1 - delta) / q) / (1 / q - theta_c *
             (R_c / R))
    ]
```

```
    return F

# Parametres initiaux
beta = 0.99
alpha = 0.4
gamma = 0.8
delta = 0.05
theta_c = 0.85
sigma = 0.33
lambd = 0.015
q = 0.5

# Plage de valeurs de theta_d
theta_d_values = np.linspace(0.2, 0.8, num=30)

variable_values_all = []

for theta_d in theta_d_values:
    equilibrium_values = fsolve(model_politique_macroprudentielle, [1,
        1, 0.14, 0.3, 0.4, 3, 1], args=(beta, alpha, gamma, delta,
        theta_c, theta_d, sigma, lambd, q))
    variable_values_all.append(equilibrium_values)

variable_values_all = np.array(variable_values_all)

# Afficher l'evolution des variables d, kappa, k_c et k_d pour chaque
    valeur de theta_d
variables = ['d', 'kappa', 'k_c', 'k_d']
fig, ax = plt.subplots(figsize=(10, 2.5))

ax.plot(theta_d_values, variable_values_all[:, 0], label='d')
ax.set_ylabel('d')
ax.legend()
ax.grid()

ax.set_xlabel('Value of theta_d')
plt.tight_layout()
plt.show()
```

```python

variables = ['kappa', 'k_c', 'k_d']
fig, axes = plt.subplots(len(variables), 1, figsize=(10, 6), sharex=True
    )

for i, ax in enumerate(axes):
    ax.plot(theta_d_values, variable_values_all[:, 4 + i], label=
        variables[i])
    ax.set_ylabel(variables[i])
    ax.legend()
    ax.grid()

axes[-1].set_xlabel('Value of theta_d')
plt.tight_layout()
plt.show()
```

Listing 2: Python code showing the impact of an increase of θ_d on the steady state

11.7 Unsaturated credit constraint case

We notice that the optimization program of entrepreneurs has a recursive form, we can thus form a dynamic Lagrangian as follows:

$$\begin{aligned}\forall t \quad L_t^E(C_t^E, B_t, S_t, K_t^c, K_t^d) = \gamma^t[\ln(C_t^E) + \sqrt{S_t}] \\ + P_t[R_t^c K_t^c + R_t^d K_t^d + B_t - R_t B_{t-1} - C_t^E - i_t^c - i_t^d] \\ + \mu_t^c[q \cdot i_t^c + (1-\delta)K_t^c - K_{t+1}^c] \\ + \mu_t^d[i_t^d + (1-\delta)K_t^d - K_{t+1}^d] \\ + \eta_t[\theta_c R_t^C K_t^c + \theta_d R_t^d K_t^d - R_{t+1} B_t]\end{aligned}$$

where $\eta_t = 0$ because the credit constraint is not saturated.

This new model therefore has the following first-order conditions :

$$D_t = \beta \cdot R_t \cdot D_{t-1} \tag{74}$$

$$C_t^H = \frac{(1-\beta)}{(1-\chi)} \cdot R_t \cdot D_{t-1} \tag{75}$$

$$\kappa_t = \frac{(K_t^c)^\sigma}{(K_t^c)^\sigma + (K_t^d)^\sigma} \tag{76}$$

$$w_t = (1-\alpha) \cdot \frac{Y_t}{L_t} \tag{77}$$

$$R_t^d = \alpha(1-\kappa_t) \cdot \frac{Y_t}{K_t^d} \tag{78}$$

$$R_t^c = \alpha \cdot \kappa_t \cdot \frac{Y_t}{K_t^c} \tag{79}$$

$$\frac{1}{R_{t+1}^d + 1 - \delta} = \frac{1/q}{R_{t+1}^c + (1-\delta)/q} \tag{80}$$

$$C_t^E + K_{t+1}^d - (1-\delta)K_t^d + \frac{K_{t+1}^c - (1-\delta)K_t^c}{q} = R_t^c K_t^c + R_t^d K_t^d + D_t - \frac{D_t}{\beta} \tag{81}$$

$$Y_t = C_t^E + C_t^H + K_{t+1}^d - (1-\delta)K_t^d + \frac{K_{t+1}^c - (1-\delta)K_t^c}{q} \tag{82}$$

Furthermore, corollary 3 tells us that :

$$R_{t+1} = R_{t+1}^d + 1 - \delta = R_{t+1}^c + \frac{1+\delta}{q} \tag{73}$$

We can therefore take equations (73), (74), (77), (78) and (79), deflate them in a similar way to the model with the introduction of macroprudential policy, and implement them on dynare.

11.8 Python code to obtain eigenvalue number 1 as a function of q

```python
import numpy as np
import matplotlib.pyplot as plt

def VP_val_1(q=6):
    beta = 0.99
```

```
    alpha = 0.4
    gamma = 0.8
    delta = 0.05
    theta_c = 0.85
    theta_d = 0.1
    sigma = 0.33
    lambd = 0.015

    R_ss = (1 + lambd) / beta
    R_c_ss = (1 / q * ((1 + lambd) / gamma - (1 - delta))) / (1 -
        theta_c + theta_c * beta / gamma)
    R_d_ss = ((1 + lambd) / gamma - (1 - delta)) / (1 - theta_d +
        theta_d * beta / gamma)
    omega_ss = (q * (1 - theta_c + theta_c * beta / gamma)) / (1 -
        theta_d + theta_d * beta / gamma)
    kappa_ss = 1 / (1 + omega_ss ** (sigma / (sigma - 1)))
    k_c_ss = alpha / ((1 + omega_ss ** (sigma / (sigma - 1))) ** ((sigma
         - alpha) / sigma)) * (omega_ss * (1 - theta_d + theta_d * beta /
         gamma)) / ((1 + lambd) / gamma - (1 - delta))
    k_d_ss = ((1 - kappa_ss) / kappa_ss * R_c_ss * k_c_ss) / (R_d_ss)
    omega_1 = -sigma * kappa_ss - alpha * (1 - kappa_ss) + alpha
    omega_2 = -sigma * kappa_ss - alpha * (1 - kappa_ss) + 1
    omega_3 = (1 - alpha / sigma) * sigma * (1 - kappa_ss) + alpha - 1
    omega_4 = (1 - alpha / sigma) * sigma * (1 - kappa_ss)
    omega_5 = (theta_c * R_c_ss) / (R_ss / q - theta_c * R_c_ss) - (
        theta_d * R_d_ss) / (R_ss - theta_d * R_d_ss)
    omega_6 = ((1 - theta_d) / (R_d_ss * (1 - theta_d) + 1 - delta) +
        theta_d / (R_ss - theta_d * R_d_ss)) * R_d_ss * ((-kappa_ss / (1
        - kappa_ss) - alpha / sigma) * sigma * (1 - kappa_ss) + alpha) -
        ((1 - theta_c) / (R_c_ss * (1 - theta_c) + (1 - delta) / q) +
        theta_c / (R_ss - theta_c * R_c_ss)) * R_c_ss * ((1 - alpha /
        sigma) * sigma * (1 - kappa_ss) + alpha - 1)
    omega_7 = ((1 - theta_d) / (R_d_ss * (1 - theta_d) + 1 - delta) +
        theta_d / (R_ss - theta_d * R_d_ss)) * R_d_ss * ((-kappa_ss / (1
        - kappa_ss) - alpha / sigma) * sigma * (1 - kappa_ss) + 1) - ((1
        - theta_c) / (R_c_ss * (1 - theta_c) + (1 - delta) / q) + theta_c
         / (R_ss - theta_c * R_c_ss)) * R_c_ss * ((1 - alpha / sigma) *
        sigma * (1 - kappa_ss))
```

```
    omega_8 = (1 / q - theta_c * R_c_ss / R_ss) * k_c_ss
    omega_9 = (1 - theta_d * R_d_ss / R_ss) * k_d_ss
    omega_10 = R_c_ss * (k_c_ss * theta_c * 1 / R_ss)
    omega_11 = R_d_ss * (k_d_ss * theta_d * 1 / R_ss)
    omega_12 = 1 / R_ss * (k_c_ss * theta_c * R_c_ss + k_d_ss * theta_d
        * R_d_ss)
    omega_13 = k_c_ss * (R_c_ss * (1 - theta_c) + (1 - delta) / q)
    omega_14 = k_d_ss * (R_d_ss * (1 - theta_d) + (1 - delta))
    omega_15 = k_c_ss * (1 - theta_c) * R_c_ss
    omega_16 = k_d_ss * (1 - theta_d) * R_d_ss
    omega_17 = theta_c * R_c_ss / R_ss * k_c_ss + theta_d * R_d_ss /
        R_ss * k_d_ss
    omega_18 = theta_c * R_c_ss / R_ss * k_c_ss
    omega_19 = theta_d * R_d_ss / R_ss * k_d_ss
    omega_20 = 1 / R_ss * (theta_c * R_c_ss * k_c_ss - theta_d * R_d_ss
        * k_d_ss)
    omega_21 = (1 + lambd) / gamma * (omega_8 - omega_10 * omega_3 -
        omega_11 * omega_1 + omega_12 / omega_5 * omega_6)
    omega_22 = (1 + lambd) / gamma * (omega_9 + omega_10 * omega_4 +
        omega_11 * omega_2 - omega_12 / omega_5 * omega_7)
    omega_23 = omega_13 + omega_15 * omega_3 + omega_16 * omega_1
    omega_24 = omega_14 - omega_15 * omega_4 - omega_16 * omega_2
    omega_25 = omega_18 / omega_17 * (1 + omega_3) + omega_19 / omega_17
         * omega_1 - omega_20 / omega_17 * omega_6 / omega_5
    omega_26 = omega_19 / omega_17 * (1 + omega_2) + omega_18 / omega_17
         * omega_4 - omega_20 / omega_17 * omega_7 / omega_5
    omega_27 = omega_18 / omega_17 * (1 + omega_3) + omega_19 / omega_17
         * omega_1 - omega_20 / omega_17 * omega_6 / omega_5 + omega_6 /
        omega_5
    omega_28 = omega_19 / omega_17 * (1 + omega_2) + omega_18 / omega_17
         * omega_4 - omega_20 / omega_17 * omega_7 / omega_5 + omega_7 /
        omega_5
    J_A = (-omega_26 * omega_23 - omega_22 * omega_27) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    J_B = (-omega_26 * omega_24 + omega_22 * omega_28) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    J_C = (-omega_25 * omega_23 + omega_27 * omega_21) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
```

```python
    J_D = (-omega_24 * omega_25 - omega_28 * omega_21) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    a = 1
    b = -(J_A + J_D)
    c = J_A * J_D - J_B * J_C
    delta = b ** 2 - 4 * a * c

    VP_1 = (-b - delta ** 0.5) / (2 * a)
    VP_2 = (-b + delta ** 0.5) / (2 * a)

    return VP_1

q_values = np.linspace(0.01, 20, 100)  # Eviter q = 0 car division par
    zero

# Stocker les valeurs propres pour chaque valeur de q
VP1_values = []

# Calculer les valeurs propres pour chaque q
for q in q_values:
    VP1 = VP_val_1(q)  # Appeler la fonction avec la valeur q
    VP1_values.append(VP1)  # Ajouter la valeur retournee a la liste

# Convertir la liste en array numpy
VP1_values = np.array(VP1_values)

# Tracer les valeurs propres en fonction de q
plt.figure(figsize=(16, 10))
plt.axhline(y=-1, color='red', linestyle='--')
plt.axhline(y=1, color='red', linestyle='--')
plt.plot(q_values, VP1_values, label='VP1')
plt.xlabel('Value of q')
plt.title('First eigenvalue as a function of q')
plt.legend()
plt.grid(True)  # Ajouter la grille
plt.show()

plt.figure(figsize=(16, 10))
```

```python
plt.axhline(y=-1, color='red', linestyle='--')
plt.axhline(y=1, color='red', linestyle='--')
plt.plot(q_values, VP1_values, label='VP1')
plt.xlabel('Value of q')
plt.title('First eigenvalue as a function of q')
plt.ylim(-2, 2)  # Limiter l axe y de -5 a 5
plt.xlim(-0.5, 1)
plt.legend()
plt.grid(True)  # Ajouter la grille
plt.show()
```

Listing 3: Python code to obtain eigenvalue number 1 as a function of q

11.9 Python code to obtain eigenvalue number 2 as a function of q

```python
import numpy as np
import matplotlib.pyplot as plt

def VP_val_2(q=6):
    beta = 0.99
    alpha = 0.4
    gamma = 0.8
    delta = 0.05
    theta_c = 0.85
    theta_d = 0.1
    sigma = 0.33
    lambd = 0.015

    R_ss = (1 + lambd) / beta
    R_c_ss = (1 / q * ((1 + lambd) / gamma - (1 - delta))) / (1 -
        theta_c + theta_c * beta / gamma)
    R_d_ss = ((1 + lambd) / gamma - (1 - delta)) / (1 - theta_d +
        theta_d * beta / gamma)
    omega_ss = (q * (1 - theta_c + theta_c * beta / gamma)) / (1 -
        theta_d + theta_d * beta / gamma)
    kappa_ss = 1 / (1 + omega_ss ** (sigma / (sigma - 1)))
    k_c_ss = alpha / ((1 + omega_ss ** (sigma / (sigma - 1))) ** ((sigma
         - alpha) / sigma)) * (omega_ss * (1 - theta_d + theta_d * beta /
```

```
        gamma)) / ((1 + lambd) / gamma - (1 - delta))
    k_d_ss = ((1 - kappa_ss) / kappa_ss * R_c_ss * k_c_ss) / (R_d_ss)
    omega_1 = -sigma * kappa_ss - alpha * (1 - kappa_ss) + alpha
    omega_2 = -sigma * kappa_ss - alpha * (1 - kappa_ss) + 1
    omega_3 = (1 - alpha / sigma) * sigma * (1 - kappa_ss) + alpha - 1
    omega_4 = (1 - alpha / sigma) * sigma * (1 - kappa_ss)
    omega_5 = (theta_c * R_c_ss) / (R_ss / q - theta_c * R_c_ss) - (
        theta_d * R_d_ss) / (R_ss - theta_d * R_d_ss)
    omega_6 = ((1 - theta_d) / (R_d_ss * (1 - theta_d) + 1 - delta) +
        theta_d / (R_ss - theta_d * R_d_ss)) * R_d_ss * ((-kappa_ss / (1
        - kappa_ss) - alpha / sigma) * sigma * (1 - kappa_ss) + alpha) -
        ((1 - theta_c) / (R_c_ss * (1 - theta_c) + (1 - delta) / q) +
        theta_c / (R_ss - theta_c * R_c_ss)) * R_c_ss * ((1 - alpha /
        sigma) * sigma * (1 - kappa_ss) + alpha - 1)
    omega_7 = ((1 - theta_d) / (R_d_ss * (1 - theta_d) + 1 - delta) +
        theta_d / (R_ss - theta_d * R_d_ss)) * R_d_ss * ((-kappa_ss / (1
        - kappa_ss) - alpha / sigma) * sigma * (1 - kappa_ss) + 1) - ((1
        - theta_c) / (R_c_ss * (1 - theta_c) + (1 - delta) / q) + theta_c
         / (R_ss - theta_c * R_c_ss)) * R_c_ss * ((1 - alpha / sigma) *
        sigma * (1 - kappa_ss))
    omega_8 = (1 / q - theta_c * R_c_ss / R_ss) * k_c_ss
    omega_9 = (1 - theta_d * R_d_ss / R_ss) * k_d_ss
    omega_10 = R_c_ss * (k_c_ss * theta_c * 1 / R_ss)
    omega_11 = R_d_ss * (k_d_ss * theta_d * 1 / R_ss)
    omega_12 = 1 / R_ss * (k_c_ss * theta_c * R_c_ss + k_d_ss * theta_d
        * R_d_ss)
    omega_13 = k_c_ss * (R_c_ss * (1 - theta_c) + (1 - delta) / q)
    omega_14 = k_d_ss * (R_d_ss * (1 - theta_d) + (1 - delta))
    omega_15 = k_c_ss * (1 - theta_c) * R_c_ss
    omega_16 = k_d_ss * (1 - theta_d) * R_d_ss
    omega_17 = theta_c * R_c_ss / R_ss * k_c_ss + theta_d * R_d_ss /
        R_ss * k_d_ss
    omega_18 = theta_c * R_c_ss / R_ss * k_c_ss
    omega_19 = theta_d * R_d_ss / R_ss * k_d_ss
    omega_20 = 1 / R_ss * (theta_c * R_c_ss * k_c_ss - theta_d * R_d_ss
        * k_d_ss)
    omega_21 = (1 + lambd) / gamma * (omega_8 - omega_10 * omega_3 -
        omega_11 * omega_1 + omega_12 / omega_5 * omega_6)
```

```
    omega_22 = (1 + lambd) / gamma * (omega_9 + omega_10 * omega_4 +
        omega_11 * omega_2 - omega_12 / omega_5 * omega_7)
    omega_23 = omega_13 + omega_15 * omega_3 + omega_16 * omega_1
    omega_24 = omega_14 - omega_15 * omega_4 - omega_16 * omega_2
    omega_25 = omega_18 / omega_17 * (1 + omega_3) + omega_19 / omega_17
         * omega_1 - omega_20 / omega_17 * omega_6 / omega_5
    omega_26 = omega_19 / omega_17 * (1 + omega_2) + omega_18 / omega_17
         * omega_4 - omega_20 / omega_17 * omega_7 / omega_5
    omega_27 = omega_18 / omega_17 * (1 + omega_3) + omega_19 / omega_17
         * omega_1 - omega_20 / omega_17 * omega_6 / omega_5 + omega_6 /
        omega_5
    omega_28 = omega_19 / omega_17 * (1 + omega_2) + omega_18 / omega_17
         * omega_4 - omega_20 / omega_17 * omega_7 / omega_5 + omega_7 /
        omega_5
    J_A = (-omega_26 * omega_23 - omega_22 * omega_27) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    J_B = (-omega_26 * omega_24 + omega_22 * omega_28) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    J_C = (-omega_25 * omega_23 + omega_27 * omega_21) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    J_D = (-omega_24 * omega_25 - omega_28 * omega_21) / (-omega_26 *
        omega_21 - omega_25 * omega_22)
    a = 1
    b = -(J_A + J_D)
    c = J_A * J_D - J_B * J_C
    delta = b ** 2 - 4 * a * c

    VP_1 = (-b - delta ** 0.5) / (2 * a)
    VP_2 = (-b + delta ** 0.5) / (2 * a)

    return VP_2

q_values = np.linspace(0.01, 10, 100)  # Eviter q = 0 car division par
    zero

# Stocker les valeurs propres pour chaque valeur de q
VP2_values = []

```

```
# Calculer les valeurs propres pour chaque q
for q in q_values:
    VP2 = VP_val_2(q)  # Appeler la fonction avec la valeur q
    VP2_values.append(VP2)  # Ajouter la valeur retournee a la liste

# Convertir la liste en array numpy
VP2_values = np.array(VP2_values)

# Tracer les valeurs propres en fonction de q
plt.figure(figsize=(16, 10))
plt.axhline(y=-1, color='red', linestyle='--')
plt.axhline(y=1, color='red', linestyle='--')
plt.plot(q_values, VP2_values, label='VP2')
plt.xlabel('Value of q')
plt.title('Second eigenvalue as a function of q')
plt.legend()
plt.grid(True)  # Ajouter la grille
plt.show()
```

Listing 4: Python code to obtain eigenvalue number 2 as a function of q

11.10 Python code showing the environmental disaster

```
import pandas as pd
import matplotlib.pyplot as plt

def croissance_capital_et_qualite_environnementale(nb_periodes=150,
   delta=0.05, lambd=0.04, zeta=0.17):
    """
    Calcule la croissance du capital polluant et la qualite de l'
        environnement.

    Args:
        nb_periodes (int): Nombre de periodes a etudier
        delta (float): Taux de depreciation du capital
        lambd (float): Taux de regeneration du stock environnemental
        zeta (float): Taux de depreciation du stock environnemental du a
            la pollution provenant du stock de capital polluant
```

```

    Returns:
        DataFrame: Les valeurs du capital polluant et de la qualite de l'environnement pour chaque periode
    """
    K_0_d = 15.1162
    S_0 = 100

    K_t_2 = pd.DataFrame({'K_t_d': [K_0_d]}, index=[0])
    S_t = pd.DataFrame({'S_t': [S_0]}, index=[0])
    X_t = pd.DataFrame({'X_t': [0]}, index=[0])  # Initialisation de l'axe des abscisses

    for t in range(1, nb_periodes + 1):
        K_0_d = K_0_d * 1.015  # Croissance de 1.5%
        K_t_2.loc[t] = K_0_d

        if S_0 > 0:
            S_0 = S_0 * (1 + lambd) - zeta * K_0_d
            if S_0 < 0:
                S_0 = 0  # Assure que S_0 ne devient pas negatif
        S_t.loc[t] = S_0

        X_t.loc[t] = 0  # Valeurs constantes pour l'axe des abscisses

    # Affichage des graphiques
    plt.figure(figsize=(15, 8))
    plt.plot(X_t, label='Abscissa axis', color='red', linestyle='--')
    plt.plot(K_t_2, label='$K_{t\_d}$', color='blue')
    plt.plot(S_t, label='$S_{t}$', color='green')
    plt.xlabel('Periods')
    plt.title('Growth in polluting capital and environmental quality')
    plt.legend()
    plt.show()


    return pd.concat([K_t_2, S_t], axis=1)

```

```
# Appel de la fonction pour afficher les resultats
croissance_capital_et_qualite_environnementale()
```

Listing 5: Python code : Growth in polluting capital and effect on environmental quality